Galileo AltBOC E5 signal characteristics for optimal tracking algorithms

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Agenda

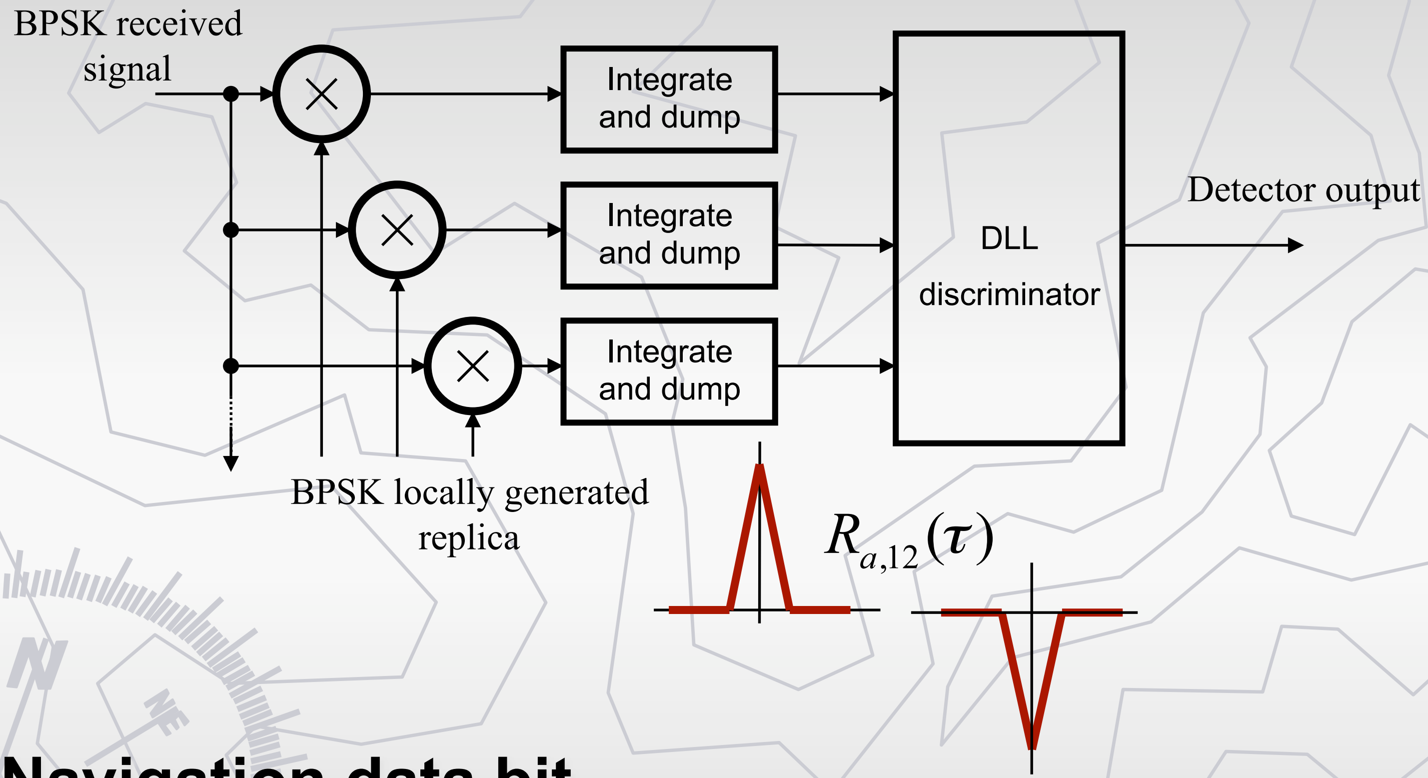
- Motivation
- Theoretical background
- Derivation of AltBOC signal ACF/CCF formula
- Result – formula usage
- Conclusion

Motivation

- BPSK signal tracking
- AltBOC signal proccesing – comparison with BPSK

Motivation

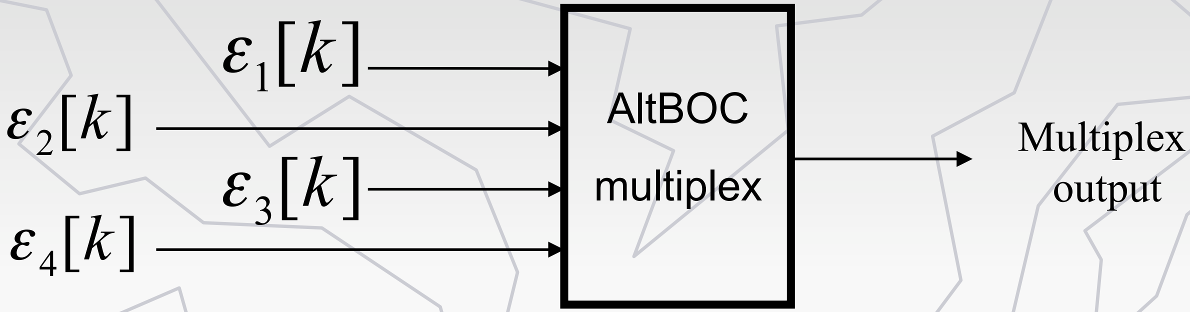
BPSK signal tracking



- **Navigation data bit**
  - **Correct bit synchronization (data bit transition can not be involved into integration interval)**
  - **Can be easily suppressed with non-coherent DLL detector**

Motivation

AltBOC signal processing



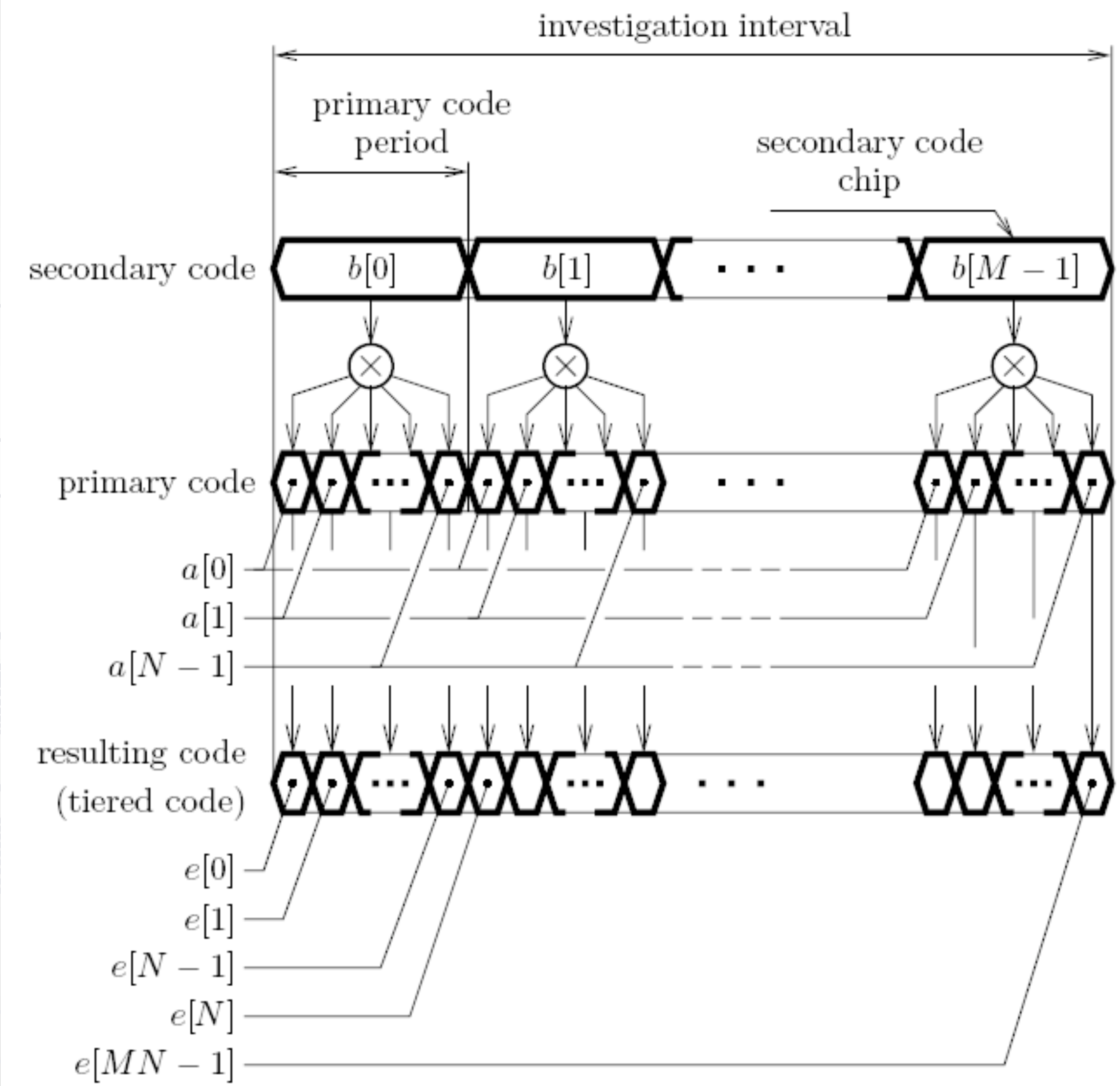
- **Four multiplex inputs: pilot signals & data signals**
  - **ACF/CCF depends on navigation data bits and secondary sequences**
- **Tracking can not be solved similarly as BPSK**
- **Goal: finding suitable description of AltBOC signal ACF/CCF**

Theoretical background

- Tiered sequences construction and their correlations property

Theoretical background

Tiered sequences – construction



Tiered sequences – properties

- CCF of tiered sequence
  - Can be computed from primary seq. CCF and secondary seq. CCF
  - Sequences can be investigated separately in tiers
  - Can be generalize for arbitrary number of tiers

$$\rho_{e,12}[m] = \sum_{l=-\infty}^{\infty} \rho_{b,12}[l]\, \rho_{a,12}[m - lN]$$

$$\mathcal{R}_{e,12}[m] = \frac{1}{N} \sum_{l=-\infty}^{\infty} \mathcal{R}_{b,12}[l]\, \rho_{a,12}[m - lN]$$

AltBOC signal definition

- Defined in Galileo ICD
  - AltBOC multiplex has four inputs $\varepsilon_1[k],\ldots,\varepsilon_4[k]$
  - Subcarriers $sc_s[k], sc_s[k-2], sc_p[k], sc_p[k-2]$

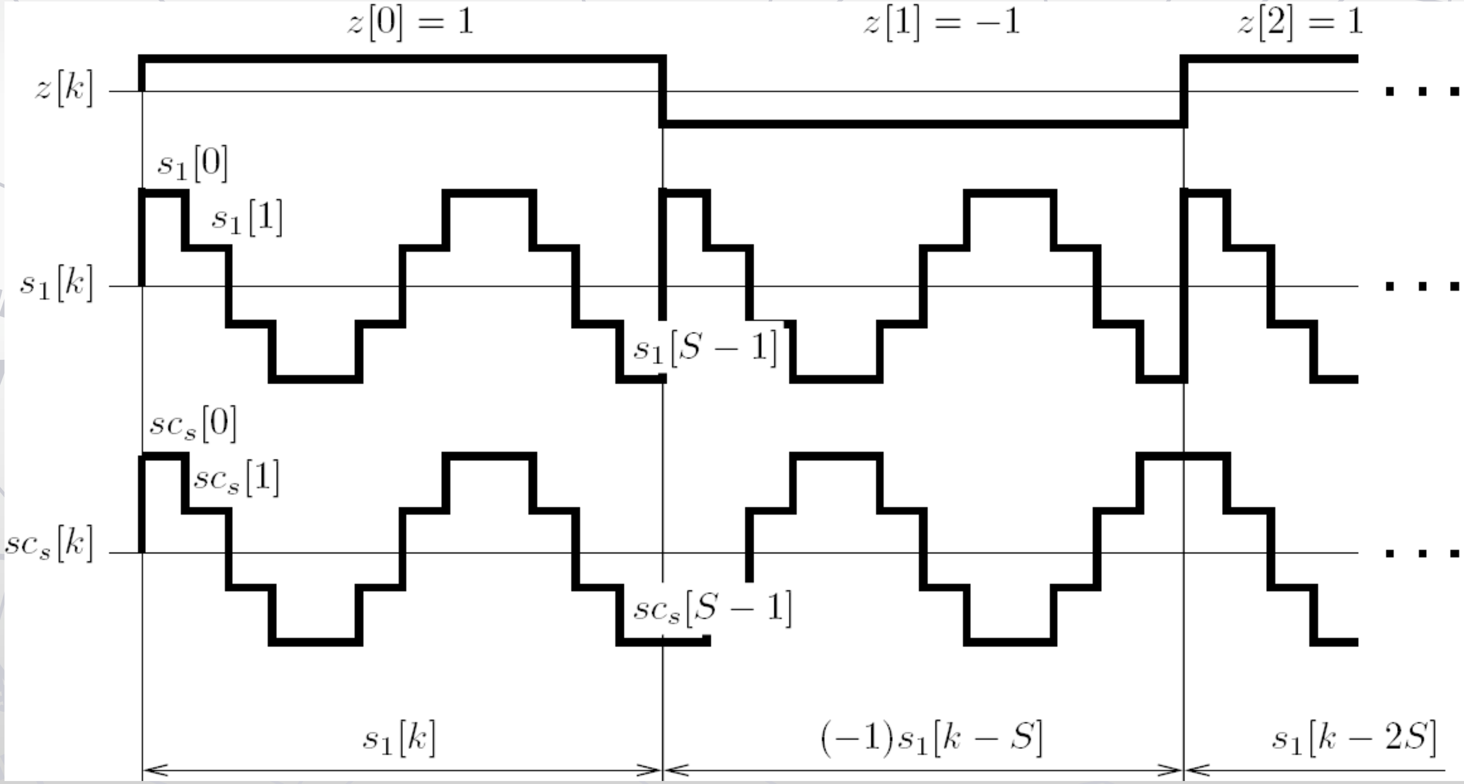
$$\mathcal{E}5[k] = \frac{\sqrt{P}}{2\sqrt{2}} \times$$
$$\Big\{ \left(\varepsilon_1[k] + \mathrm{j}\varepsilon_2[k]\right)\left(sc_s[k] - \mathrm{j}sc_s[k-2]\right) +$$
$$+ \left(\varepsilon_3[k] + \mathrm{j}\varepsilon_4[k]\right)\left(sc_s[k] + \mathrm{j}sc_s[k-2]\right) +$$
$$+ \left(\overline{\varepsilon}_1[k] + \mathrm{j}\overline{\varepsilon}_2[k]\right)\left(sc_p[k] - \mathrm{j}sc_p[k-2]\right) +$$
$$+ \left(\overline{\varepsilon}_3[k] + \mathrm{j}\overline{\varepsilon}_4[k]\right)\left(sc_p[k] + \mathrm{j}sc_p[k-2]\right) \Big\}$$

Outline of derivation of AltBOC signal ACF/CCF

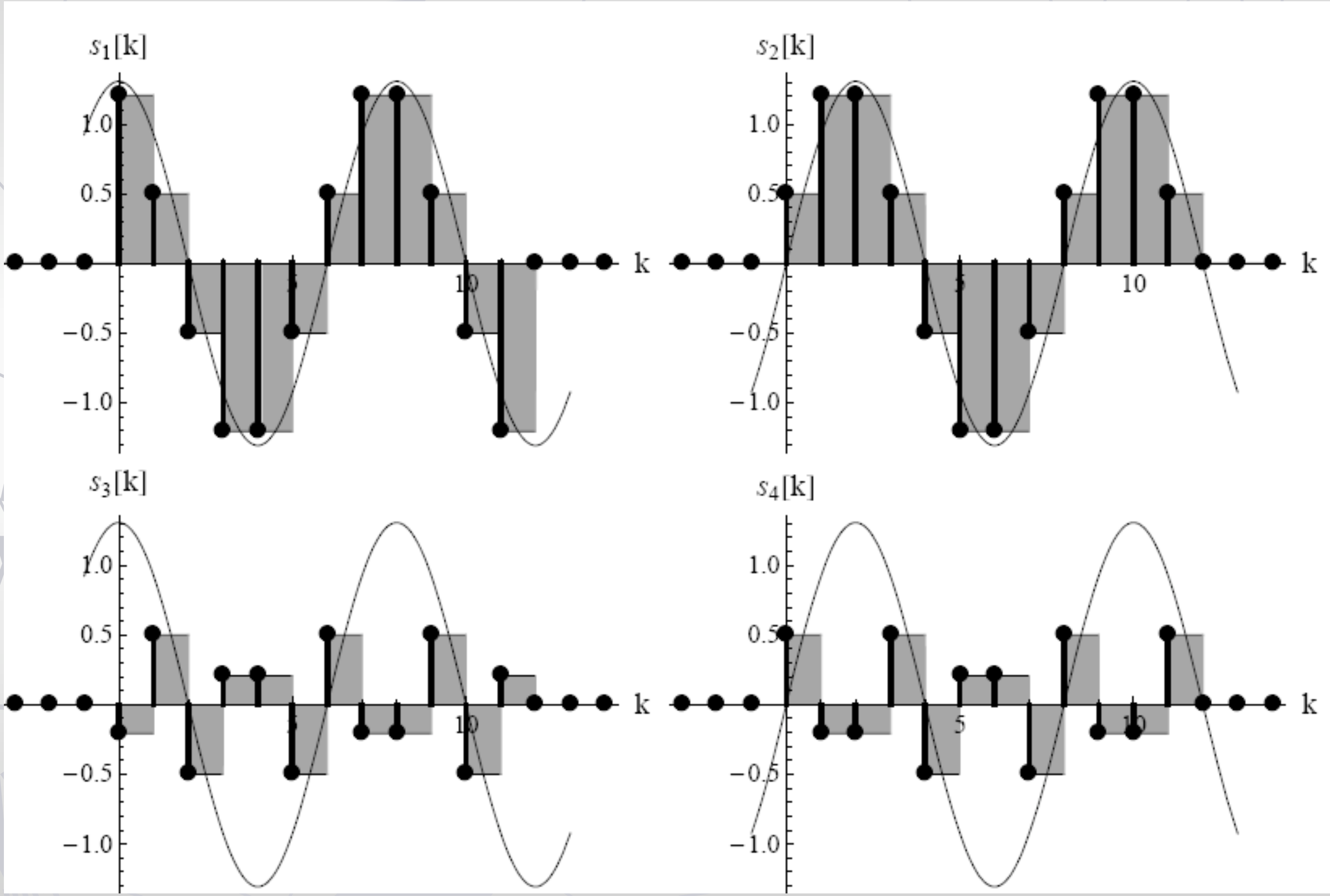
- Expression the Galileo received signal and the locally generated replica in several tiers
  - AltBOC multiplex inputs – done
  - AltBOC subcarrier – has to be solved
- Each tiers can be solved separately

Subcarrier decomposition

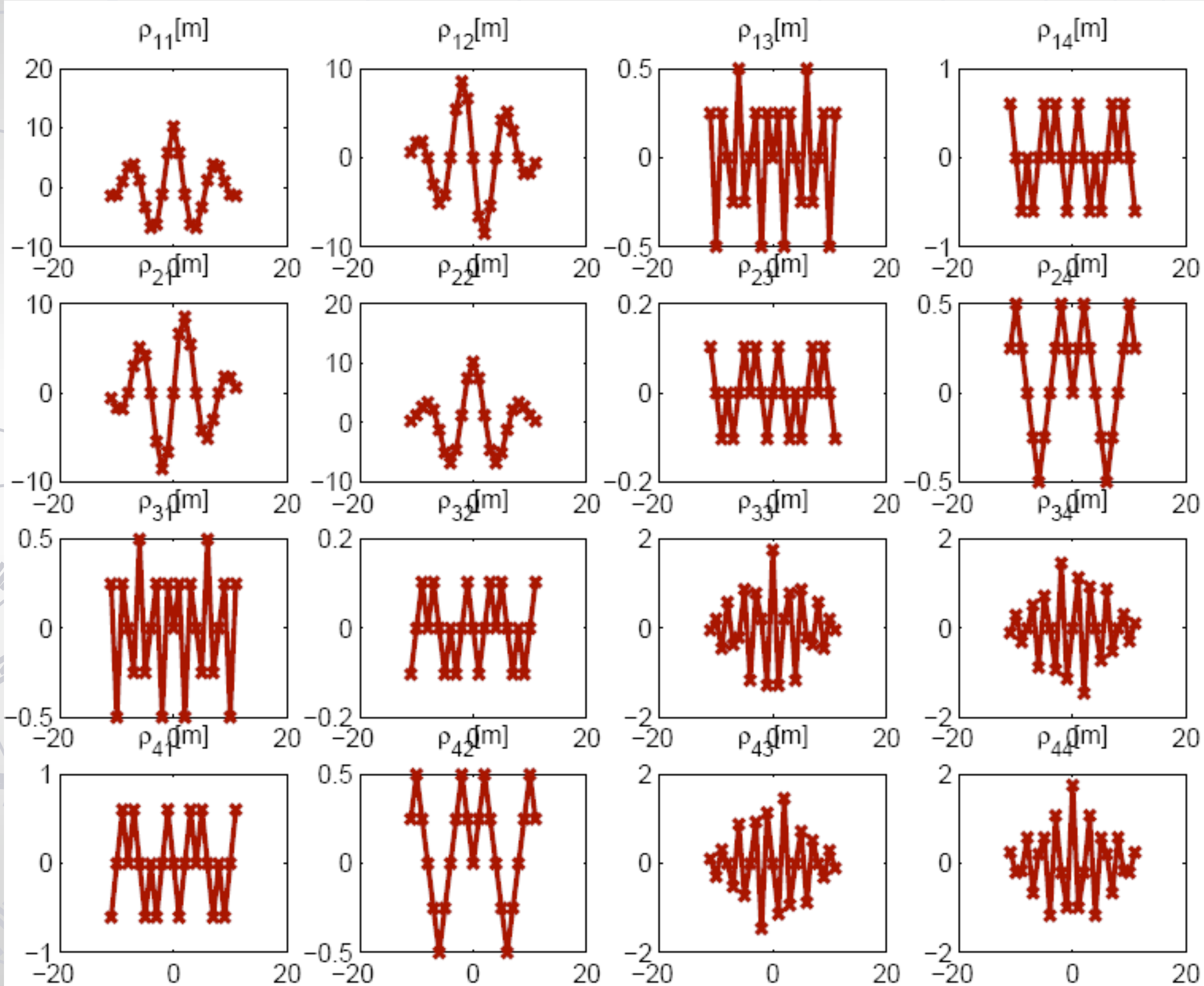
- Subcarrier decomposition into the finite sequence $s_i[k]$ and the sign sequence $z[k]$
- Properties of tiered sequences can be used for CCF investigation



Finite subcarrier sequences



Subcarrier CCF/ACF



Final formula

- **Real part:**

$$\mathrm{Re}\{\mathcal{R}_{{}^{s}\mathcal{E}5,{}^{r}\mathcal{E}5}[m]\} = \frac{\sqrt{{}^{s}P}\sqrt{{}^{r}P}}{8}\frac{1}{S} \times \Big\{$$

$$\mathrm{Im}\{\mathcal{R}_{{}^{s}\mathcal{E}5,{}^{r}\mathcal{E}5}[m]\} = \frac{\sqrt{{}^{s}P}\sqrt{{}^{r}P}}{4}\frac{1}{S} \times \Big\{$$

$$\sum_{j=-\infty}^{\infty} \Big(\mathcal{R}_{{}^{s}b_1,{}^{r}b_1}[j] + \mathcal{R}_{{}^{s}b_2,{}^{r}b_2}[j] - \mathcal{R}_{{}^{s}b_3,{}^{r}b_3}[j] - \mathcal{R}_{{}^{s}b_4,{}^{r}b_4}[j]\Big) \times$$

$$\times\ \rho_{s,12}[m - jNS] +$$

$$\sum_{j=-\infty}^{\infty} \Big(\mathcal{R}_{{}^{s}\bar{b}_1,{}^{r}\bar{b}_1}[j] + \mathcal{R}_{{}^{s}\bar{b}_2,{}^{r}\bar{b}_2}[j] - \mathcal{R}_{{}^{s}\bar{b}_3,{}^{r}\bar{b}_3}[j] - \mathcal{R}_{{}^{s}\bar{b}_4,{}^{r}\bar{b}_4}[j]\Big) \times$$

$$\times\ \rho_{s,34}[m - jNS]\Big\}$$

Final formula

- Formula for CCF of Galileo E5 AltBOC signal depends
  - ACF/CCF of finite subcarriers $s_i[k]$ – were computed; provided in tabular and graphics form
  - CCF of secondary sequences in received signal and locally generated replica – short: can be computed easily

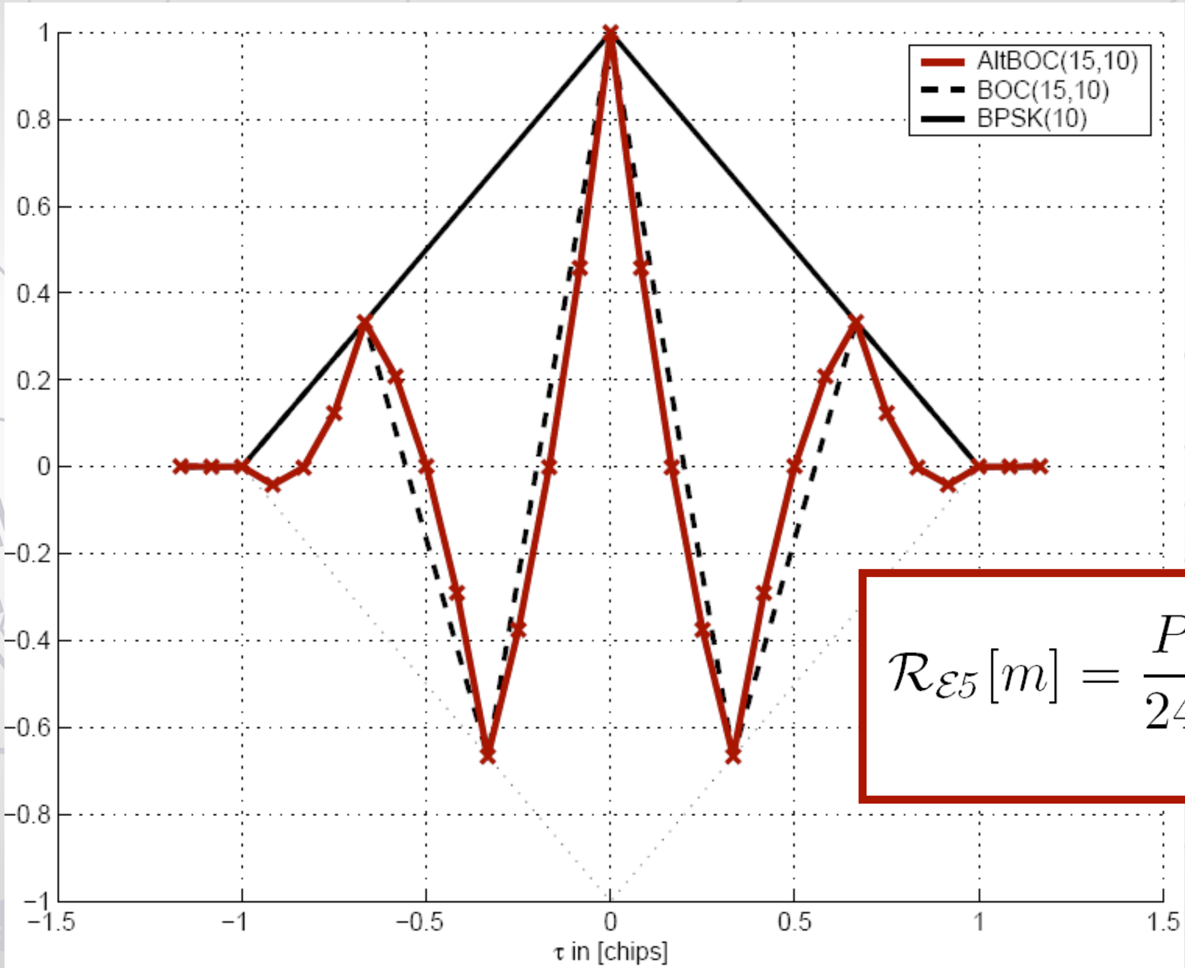
Results

How to use the formula – special cases

- ACF of AltBOC modulated signal
- CCF 1ms integration time
  - before secondary sequences synchronization: 16 possibilities
  - after secondary sequences synchronization: 4 possibilities

Results

ACF of AltBOC modulated signal

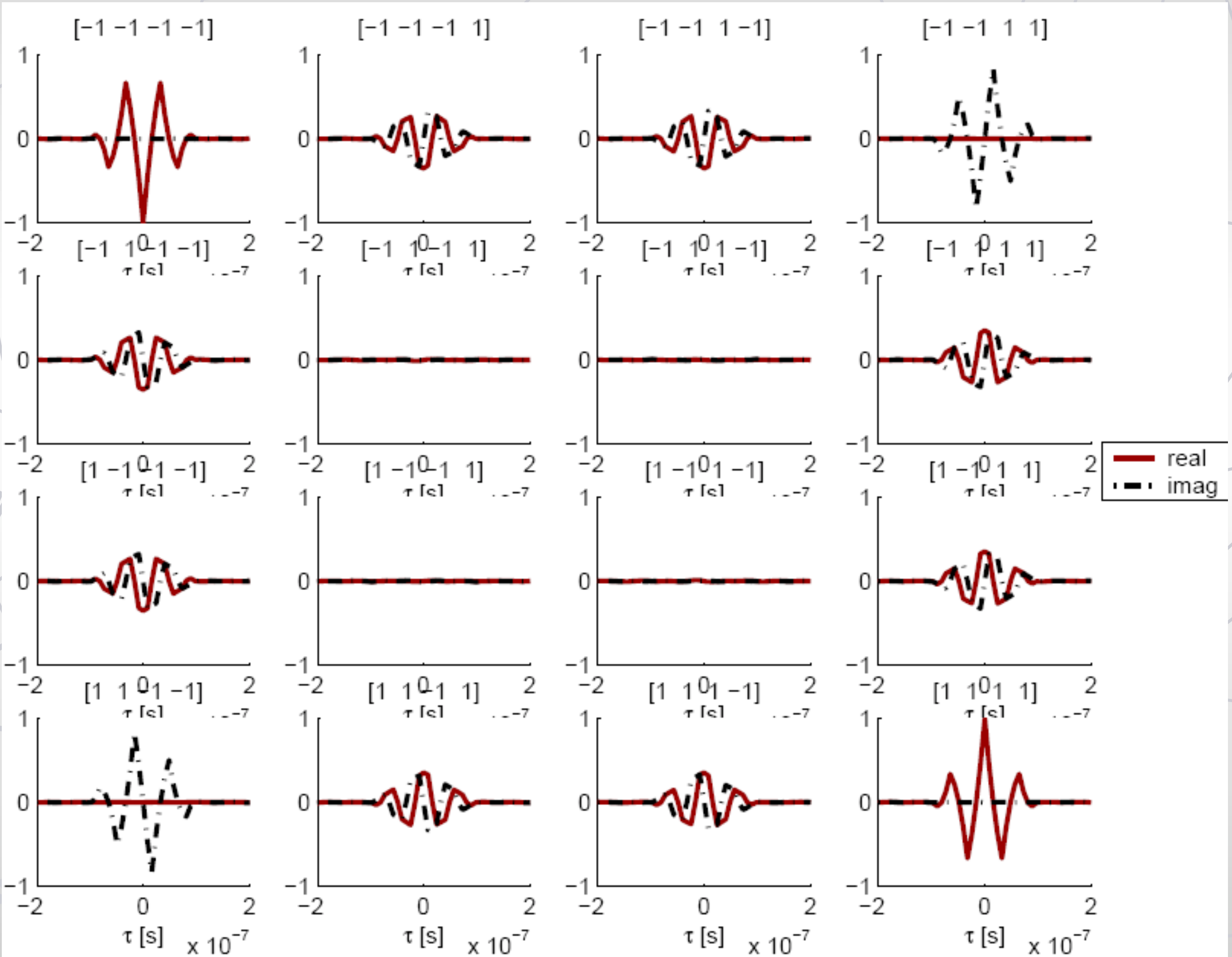


$$\mathcal{R}_{\mathcal{E}5}[m] = \frac{P}{24} \sum_{j=-\infty}^{\infty} \sum_{i=1}^{4} \rho_{s,ii}[m - jNS]$$

Results

1ms integration interval

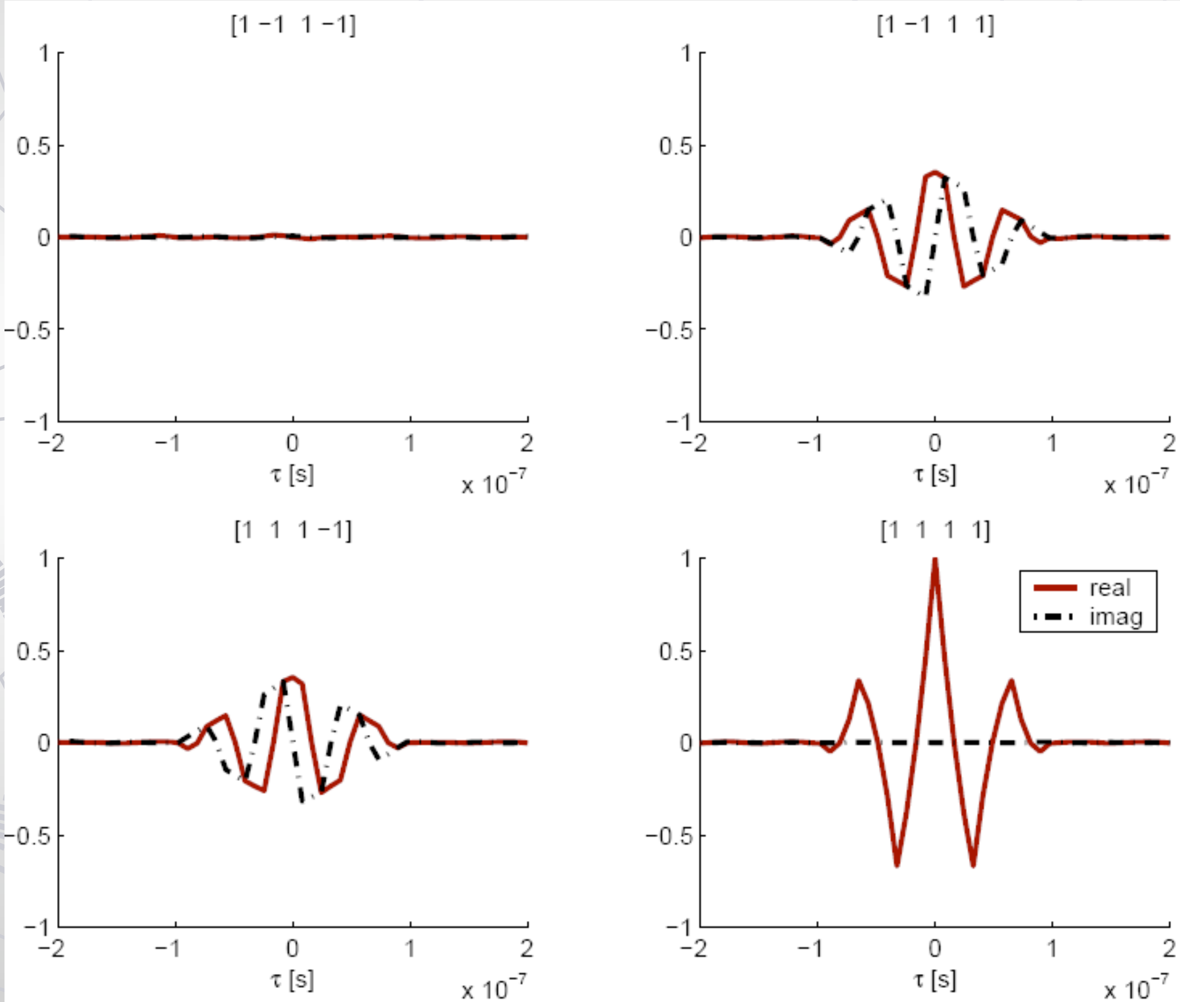
- before secondary sequences synchronization: 16 possibilities



Results

1ms integration interval

- after secondary sequences synchronization: 4 possibilities



Conclusions

- The problem of the AltBOC signal tracking was identified
- Analytical formula of CCF of Galileo E5 AltBOC was derived
- The derivation based on signal decomposition in several tiers
- Formula depends
  - Subcarierrs ACF/CCF
  - Secondary sequences CCF

Thank you for your attention

Auto / Cross Correlation Function ACF/CCF

- Discrete time
  - Finite sequence

$$\rho_{a,12}[m] = \sum_{k=-\infty}^{\infty} a_1[k]\, a_2[k+m]$$

  - Periodic sequence

$$\mathcal{R}_{a,12}[m] = \frac{1}{N} \sum_{k=0}^{N-1} \ddot{a}_1[k]\, \ddot{a}_2[k+m]$$

- Continuous time
  - Finite signal

$$\rho_{a,12}(\tau) = \int_{-\infty}^{\infty} a_1(t)\, a_2(t+\tau)\, \mathrm{d}t$$

  - Periodic signal

$$\mathcal{R}_{a,12}(\tau) = \frac{1}{NT_c} \int_{0}^{NT_c} \ddot{a}_1(t)\, \ddot{a}_2(t+\tau)\, \mathrm{d}t$$

Ideal pseudorandom sequence

- Defined with its correlation property

$$\rho_{I,ij}[m] = \begin{cases} N\delta[m] & \text{for } i = j, \\ 0 & \text{otherwise} \end{cases}$$

- Well represents all pseudorandom sequences with length *N*
- Purpose: simplify formula manipulations

Auto / Cross Corelation Function ACF/CCF

- ??

	Finite signal / sequence	Periodical Signal / sequence
Sequencies (Discrete time signals)	$\rho_{a,12}[m] = = \sum_{k=-\infty}^{\infty} a_1[k]a_1[k+m]$	$R_{a,12}[m] = = \frac{1}{N}\sum_{k=0}^{N-1} \ddot{a}_1[k]\ddot{a}_2[k+m]$
Continuous time signals	$R_{a,12}(\tau) = = \int_{-\infty}^{\infty} a_1(t)a_2(t+\tau)dt$	$R_{a,12}(\tau) = = \frac{1}{NT_c}\int_{0}^{NT_c} \ddot{a}_1(t)\ddot{a}_2(t+\tau)dt$

Ideal pseudorandom sequence

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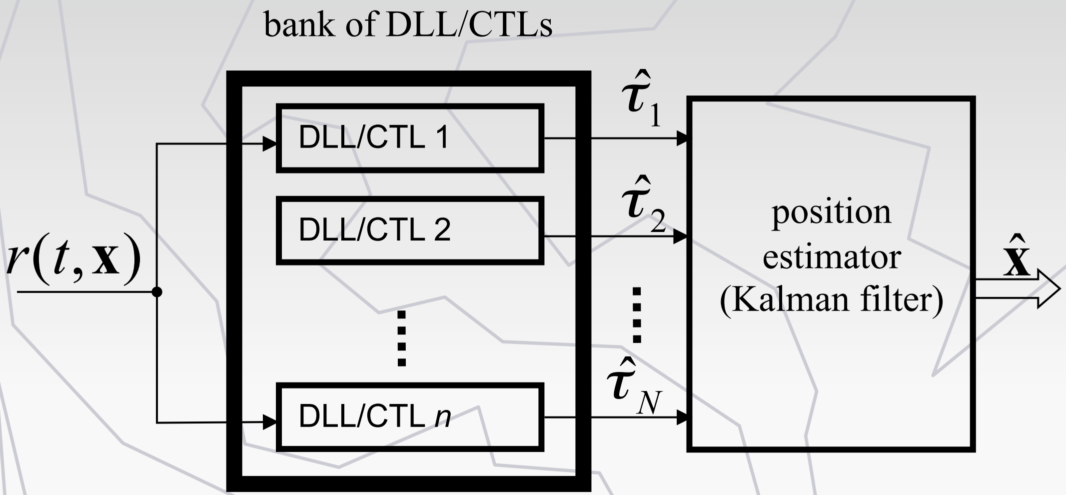
GNSS receiver



Contemporray GNSS receiver structure

- estimation of *N* pseudoranges

- position estimation based on independent pseudoranges

- **GNSS receiver – Time of Arrived Conception – pseudoranges measurement (possible due to suitable correlation property of signals)**
- **Core of navigation receiver – bank of tracking loops (combinations of CTLs and DLLs) ⇨ pseudoranges**

Baseband signal chars. I
Description

- **Discrete time**
  - **Finite signal**

$$\alpha[k] = \begin{cases} a[k] & k \in 0,\dots,N-1 \\ 0 & \text{otherwise} \end{cases}$$

  - **Periodic signal**

$$a[k] = \sum_{m=-\infty}^{\infty} \alpha(k - mN)$$

- **Continuous time**
  - **Finite signal**

$$h(t) = \begin{cases} c(t) & t \in \langle 0, NT_c) \\ 0 & \text{otherwise} \end{cases}$$

  - **Periodic signal**

$$c(t) = \sum_{m=-\infty}^{\infty} h(t - mNT_c)$$

Baseband signal chars. II
Time domain chars. – ACF

- **Discrete time**
  - **Finite signal**

$$\rho_\alpha[m] = \sum_{k=-\infty}^{\infty} \alpha[k]\alpha[k+m]$$

  - **Periodic signal**

$$R_a[m] = \frac{1}{N}\sum_{k=0}^{N-1} a[k]a[k+m]$$

- **Continuous time**
  - **Finite signal**

$$\rho_h(\tau) = \int_0^{NT_c} h(t)h(t+\tau)\,\mathrm{d}t$$

  - **Periodic signal**

$$R_c(\tau) = \frac{1}{NT_c}\int_0^{NT_c} c(t)\,c(t+\tau)\,\mathrm{d}t$$