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A CONING COMPENSATION ALGORITHM WITH PURE FILTERED ANGLE RATE INPUT

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1 Introduction –where I from?

NRC

College of Automation Engineering

Campus of NUAA

The Navigation Research Center (NRC), College of Automation Engineering ,Nanjing University of Aeronautics and Astronautics, is engaged in the research of navigation and traffic technology.

1 Introduction –NRC staff

Subdean of Department

senior engineer

Subdecanal of college

Jianye Liu
Head of college

grand old man

Member of China

Associate professors

1 Introduction –NRC focus

The researches of NRC focus on:

Modern Inertial Navigation System & GNSS.

New Inertial Integrated Navigation Theory and Application.

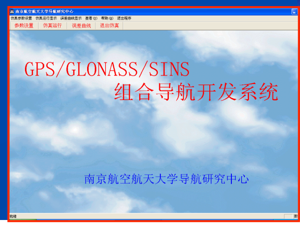
Positioning with Wireless sensor network.

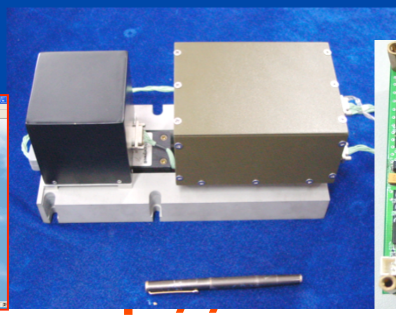
Mini Low Cost Navigation System for Vehicle

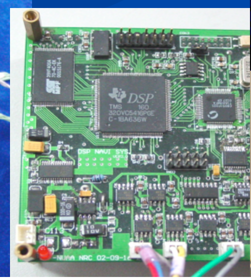
Integrated positioning & navigation system.

GIS application

and so on.







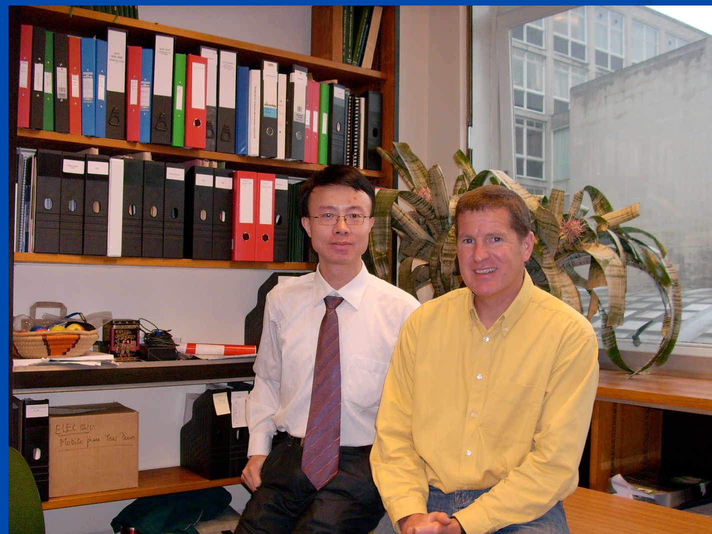
1 Introduction –myself

tsinghua

With the help of UK&Chinese scientists, I was awarded a full scholarship under the UK/China Scholarships for Excellence programme as the visiting researcher in Leeds University in 2007.

Thank you very much.

Leeds University



With Dr Andrew Kemp

1 Introduction-what is navigation

wikipedia

Navigation is the process of reading, and controlling the movement of a craft or vehicle **from one place to another**

Contents [hide]

1 Introduction-different kinds

Dead reckoning or DR, inertial navigator

Pilotage

Celestial navigation

Radio navigation

Radar navigation

Satellite navigation

1 Introduction-In details

Container carrier

Attitude is important!

plane

satellite

Daily application

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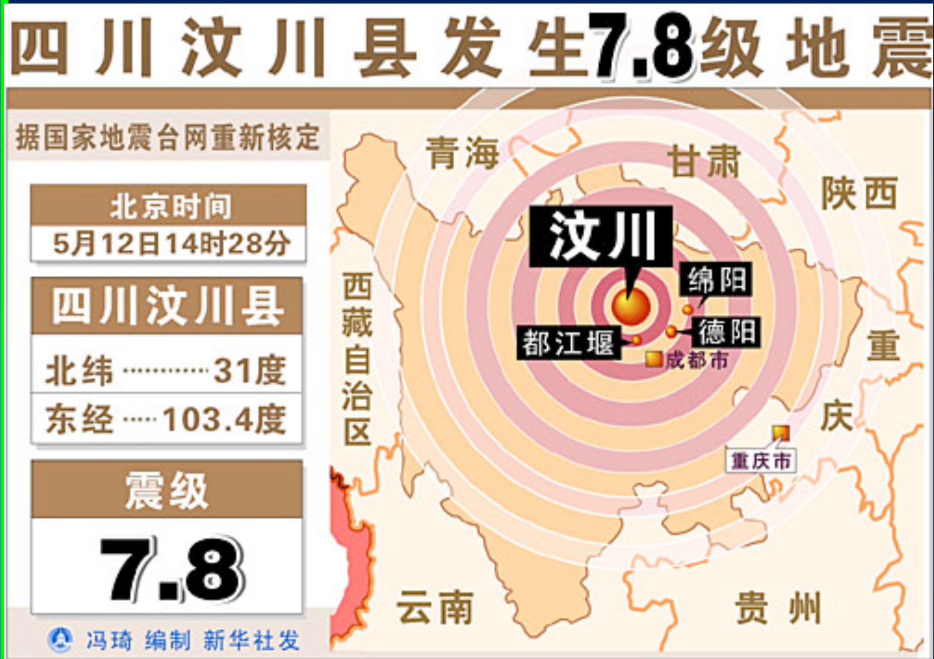
1 Introduction-In details



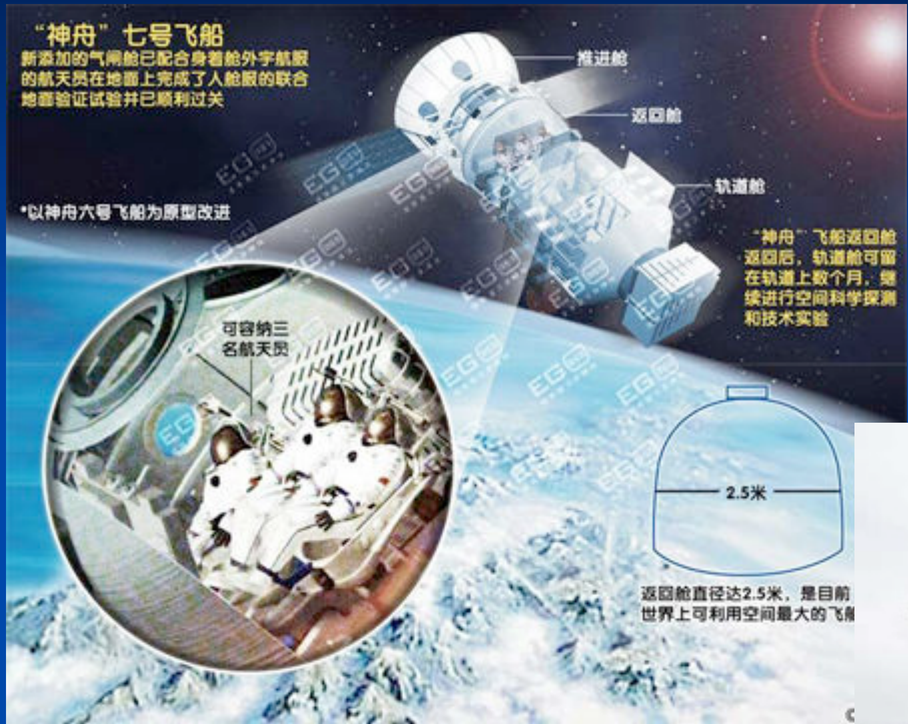
Earthquake communication

Attitude is important!

Emergent communication
With satellite

Thank you all very much for helping China during the earthquake nightmare.

1 Introduction-In details



Attitude is important!

Shenzhou 7 Launch
- China Space Walk

Measure & communicate ship
Tracking the spaceship

1 Introduction-How to get attitude

Dead reckoning or DR,
inertial navigator

Pilotage

Celestial navigation

What can supply Attitude?

Radio navigation

Radar navigation

Satellite navigation

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1 Introduction-How to get attitude

Dead reckoning or DR,
inertial navigator

It can get the accurate attitude!
How to improve the precision?

Celestial navigation

Low frequency update information.

It is hard to get attitude
1)three antenna (cost)
2)Stars might be invisible.

Satellite navigation

1 Introduction

In the Strapdown Inertial Navigation System (SINS), the **coning error** of the inertial measurement unit (IMU) is one of the **major factors** affecting SINS accuracy, especially for high precision units.

1 Introduction

Assuming in coning motion, the motion equation along corresponding axis is

$$\Phi = \left[0, \quad a\cos(\Omega t), \quad a\sin(\Omega t)\right]^T$$

Classical coning motion ：

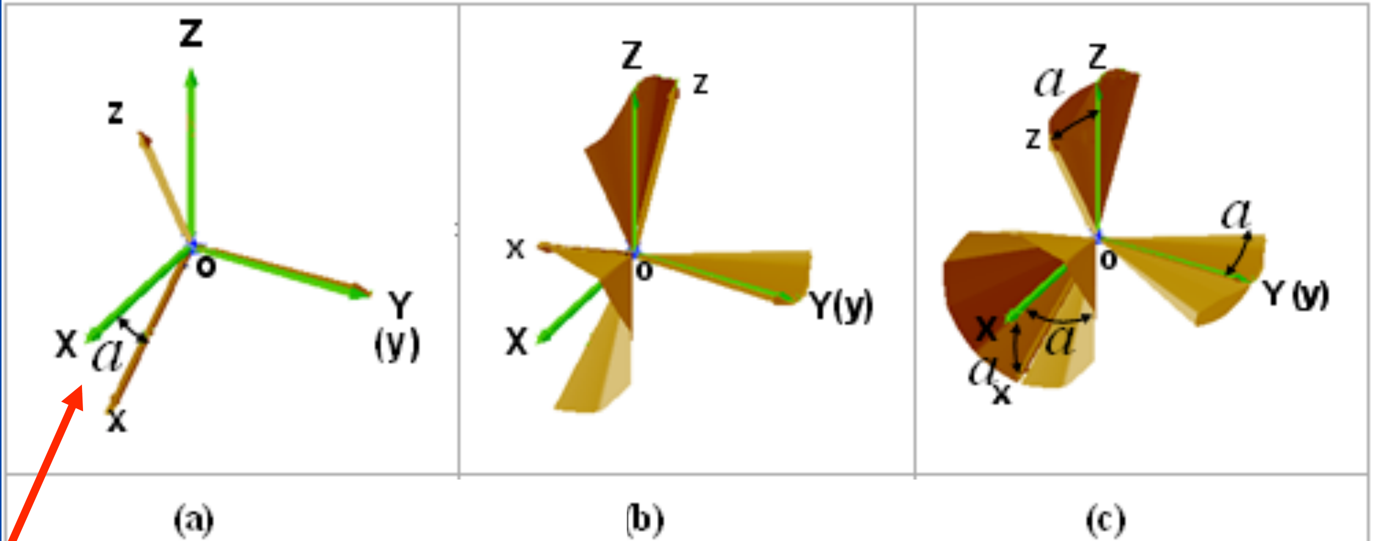

Fig of Classical coning motion

coning half-angle

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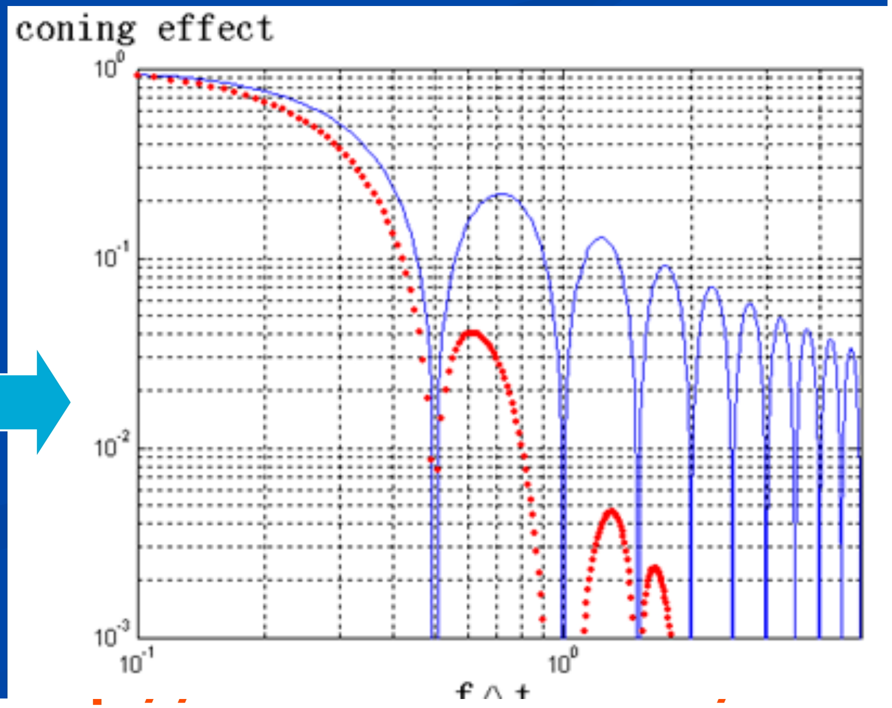
2 Filtered angular rate coning effect analysis

Substituting the filtered angular rate into the coning algorithm with ideal angular increments is unable to realize an excellent coning error compensation effect.

Difference of coning effect between the ideal physical quantity (continuous line) and project filtered one (dotted line) [8]

[Reference 12, Mark]



3 Filtered physical quantity analysis

Considering the filter function $F(\Omega)$, the filtered angular rate vector and angular increment vector

$$\boldsymbol{\omega}_{nb}^{b}(t)=\begin{bmatrix}-2F(0)\Omega\sin^2(a/2)\\ -\Omega F(\Omega)\sin(a)\sin\Omega t\\ \Omega F(\Omega)\sin(a)\cos\Omega t\end{bmatrix}$$

$$\boldsymbol{\theta}(h)=$$

$$\Omega h\begin{bmatrix}-2F(0)\sin^2(a/2)\\ -F(\Omega)\sin(a)\sin c(\Omega h/2)\sin[\Omega(t+h/2)]\\ F(\Omega)\sin(a)\sin c(\Omega h/2)\cos[\Omega(t+h/2)]\end{bmatrix}$$

What we get

The rotation vector in the body coordinate can be expressed as

$$\boldsymbol{\Phi}=\Omega h\begin{bmatrix}-2F^2(\Omega)\sin^2(a/2)\sin c(\Omega h)\\ -F(\Omega)\sin(a)\sin c(\Omega h/2)\sin[\Omega(t+h/2)]\\ F(\Omega)\sin(a)\sin c(\Omega h/2)\cos[\Omega(t+h/2)]\end{bmatrix}$$

What we want

3 Filtered physical quantity analysis

$$\Omega h \begin{bmatrix} -2F(0)\sin^2(a/2) \\ -F(\Omega)\sin(a)\sin c(\Omega h/2)\sin[\Omega(t+h/2)] \\ F(\Omega)\sin(a)\sin c(\Omega h/2)\cos[\Omega(t+h/2)] \end{bmatrix}$$

The components of angular increment and rotation vector along axis Y and Z change periodically, which will not cause the unlimited volatility of attitude angle.

The constant component along the X axis directly causes the attitude angle error, the error principle can be defined as

$$\varepsilon = \Phi_x - \hat{\theta}_x - \delta\hat{\Phi}_x$$

4 Algorithm for angular increment from filtered angular rate

gyro angular rate can be expressed

$$\omega(t+\tau) = g_0 + g_1\tau + g_2\tau^2 + \cdots + g_M\tau^M$$

the inner integration angle

$$\hat{\theta}(h) = \int_0^h \omega(t+\tau)d\tau = g_0 h + \frac{1}{2}g_1 h^2 + \frac{1}{3}g_2 h^3 + \cdots + \frac{1}{M+1}g_M h^{M+1} = HGh$$

the formula for fitting angular increment is

$$\hat{\theta}(h) = HGh = (HC^{-1})Wh = SWh = (s_0\omega_0 + s_1\omega_1 + \cdots + s_M\omega_M)\,h$$

[Reference 12]

5 Coning error coefficients of compensation algorithm with filtered angular rate input

The X-axis component of the **vector cross product** is

$$
\begin{aligned}
& C_{px}(n) \\
& = T\omega_{(nM)y} T\omega_{(nM+p)z} - T\omega_{(nM+p)y} T\omega_{(nM)z} \\
& = (\Omega T)^2 F^2(\Omega) \sin^2 \alpha \sin(p\Omega T)
\end{aligned}
$$

5 Coning error coefficients of compensation algorithm with filtered angular rate input

the axis X correction of rotation vector can be expressed as:

$$\delta\Phi_x = \sum_{p=1}^{M} C_{px} x_p$$

So We can get

$$\sin^2 a \sum_{p=1}^{M} \{ \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \lambda^{2k+1}}{(2k+1)!} [(p-1)^{2k+1} - 2p^{2k+1} + (p+1)^{2k+1}] \} x_p$$

$$= 2\lambda \sin^2 (a/2) \left\{ \sum_{k=1}^{\infty} \frac{(-1)^{k+1} \lambda^{2k}}{(2k+3)!} [(M-1)^{2k+3} - 2M^{2k+3} + (M+1)^{2k+3}] \right\}$$

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5 Coning error coefficients of compensation algorithm with filtered angular rate input

Given k=1 to M, Then M equations can be obtained as

$$\sin^2 a$$

$$\sum_{p=1}^{M}\{\frac{(-1)^{k+1}\lambda^{2k+1}}{(2k+1)!}[(p-1)^{2k+1}-2p^{2k+1}+(p+1)^{2k+1}]\}x_p$$

$$= 2\lambda\sin^2(a/2)$$

$$\left\{\frac{(-1)^{k+1}\lambda^{2k}}{(2k+3)!}[(M-1)^{2k+3}-2M^{2k+3}+(M+1)^{2k+3}]\right\}$$

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5 Coning error coefficients of compensation algorithm with filtered angular rate input

the correction coefficients of compensation algorithm

$$X_{M\times 1} = [x_1, x_2, \cdots, x_M]^T = A^{-1}B$$

The dominant term of residual error in X axis is as follows

$$\begin{aligned}\delta\tilde{\phi} &= \Phi_{cx} - \delta\hat{\Phi}_x \\ &= 2\sin^2(a/2)\left\{\frac{(-1)^M \lambda^{2M+3}}{(2M+5)!}[(M-1)^{2M+5} - 2M^{2M+5} + (M+1)^{2M+5}]\right\} \\ &\quad - \sin^2 a\left\{\sum_{p=1}^{M}\{\frac{(-1)^M \lambda^{2M+3}}{(2M+3)!}[(p-1)^{2M+3} - 2p^{2M+3} + (p+1)^{2M+3}]\}x_p\right\}\end{aligned}$$

6 Accuracy analysis and verification of coning algorithm with angular rate input

Coning algorithm precision in this paper is analyzed by comparing with the 4-order Runge-Kutta algorithm.

The $\boxed{\text{dominant term of residual error}}$ in 4-order Runge-Kutta algorithm is:

$$-a^2(\Omega h)^5/1440 = -a^2(\Omega T)^5/45 \qquad (28)$$

the coning compensation algorithm with filtered angular rate input in this paper has better precision than 4-order Runge-Kutta algorithm.

6 Accuracy analysis and verification of coning algorithm with angular rate input

Suppose the coning half-angle is 1 deg and the frequency of the coning motion coupled from body dither is 20Hz, (that is angular rate $\Omega \approx 126 \mathrm{rad/s}$). The sampling period of SINS is 1 ms. Under the east-north-up (ENU) geographical coordinate system, the pitch angle of the airplane will generate error drift resulted from coning motion.

6 Accuracy analysis and verification of coning algorithm with angular rate input

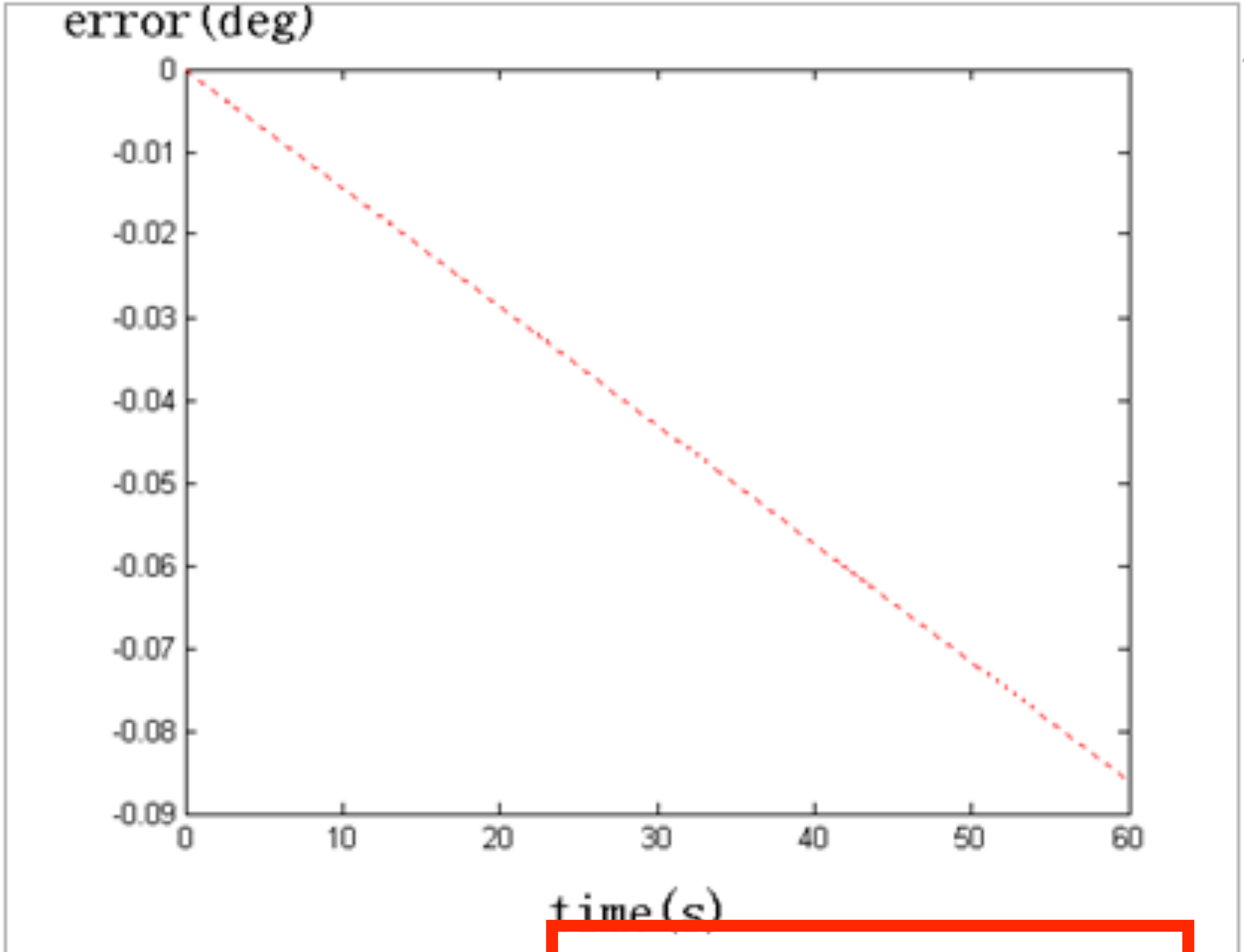

Fig 2. pitch angle error of 4 order Runge-Kutta algorithm

Large slope

Magnitude 0

6 Accuracy analysis and verification of coning algorithm with angular rate input

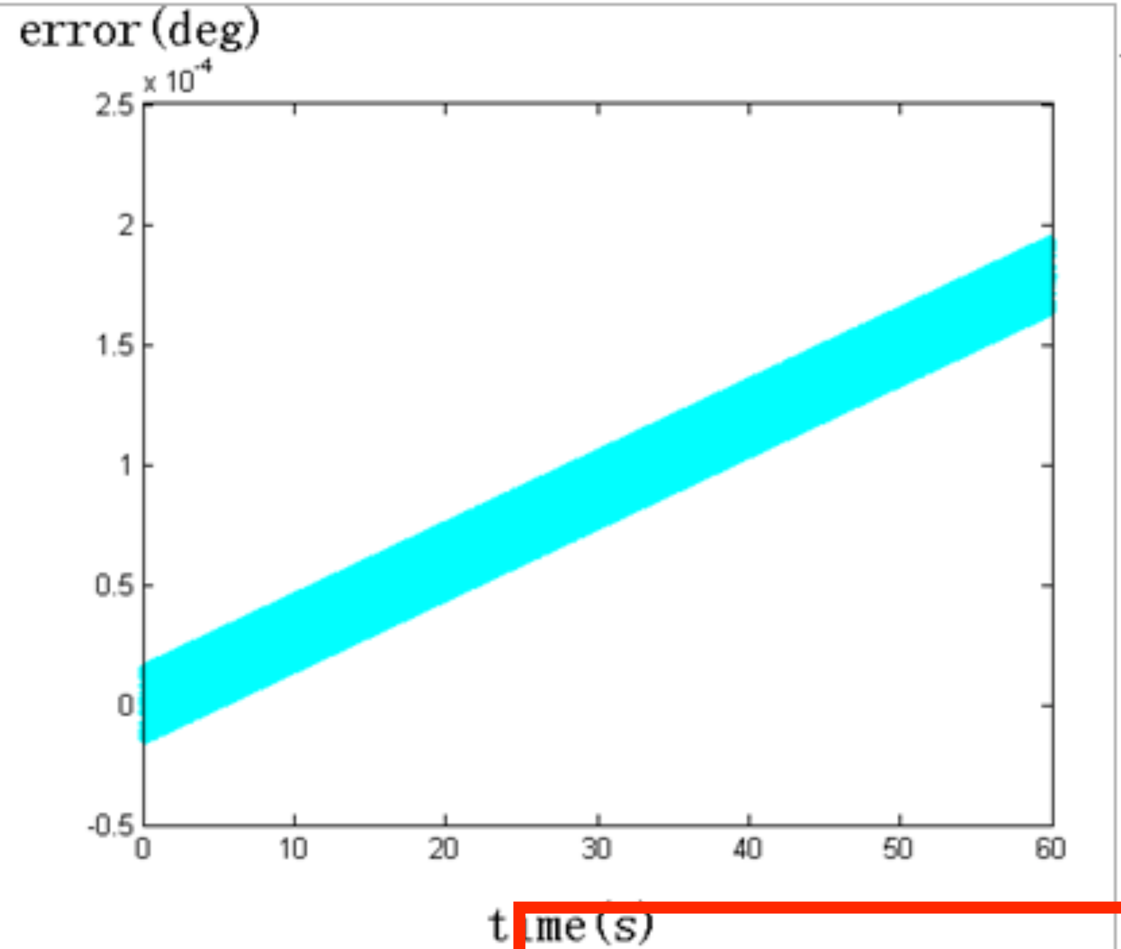

Fig 3. pitch angle error of 2 samples filtered algorithm

smaller slope

Magnitude -4

6 Accuracy analysis and verification of coning algorithm with angular rate input

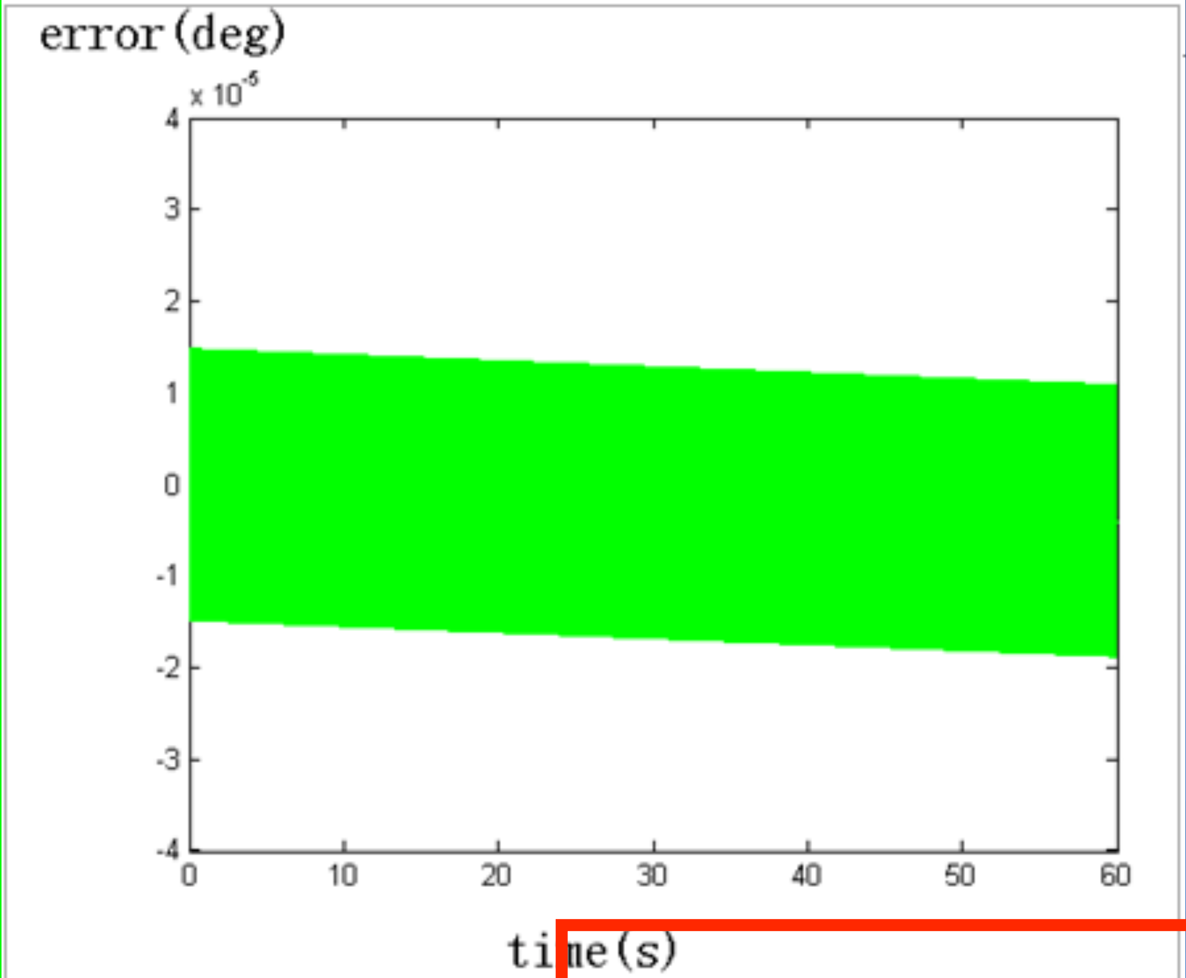

Fig 4. pitch angle error of 4 samples filtered algorithm

lowest slope

Magnitude -5

6 Accuracy analysis and verification of coning algorithm with angular rate input

1) 2-sample algorithm in this paper will improve the precision by two more orders of magnitude than 4-order Runge-Kutta algorithm, **even one more order in magnitude than the improved 2-sample algorithm mentioned in reference [11].**

2) The accuracy of 4-sample algorithm is higher than that of the 2-sample algorithm.

7 Conclusion

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Navigation? → Attitude is Very important!

→ Who can supply Attitude? → How to improve Attitude?

→ Error Analysis → Coning on one of the axis.

→ Look for the proper parameter → Simulation of the result → …

7 Conclusion

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The coning compensation algorithm with filtered angular rate input has **better performance** than other kinds of attitude algorithms, and it is good for improving the accuracy of engineering SINS.

The **following research will further analyze 2-sample and 4-sample algorithms with filtered angular rate input, especially in engineering prototype FOG SINS.**

Thank you

Welcome to visit NRC, NUAA, Nanjing, China.

Welcome to collaborate with NRC.

南京航空航天大学