## NONLINEAR SAR/INS INTEGRATION USING SIGMA-POINT KALMAN FILTER

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## ABSTRACT

Common navigation systems rely on different sensors, which are combined in the navigation filter. An inertial navigation system provides accurate relative position updates along short periods of time. To ensure long term accurate and autonomous navigation information a radar altimeter or synthetic aperture radar (SAR) can be used. Synthetic aperture sensor captures the earth surface in range and azimuth coordinates. The captured image is transformed into earth coordinates and matched to a map which contains specific well visible and unambiguous features like crossroads or rivers. The position shift between the captured SAR image and the reference feature is used to aid the navigation information.

Due to the nonlinearity of the measurement equation sigma-point Kalman filter was implemented for SAR/INS integration. Higher order terms in the measurement equation may become significant in the presence of larger height errors. Sigma-point Kalman filter (SPKF) takes higher order terms into consideration.

This paper concentrates on the processing of the detected position shift between the SAR image and the reference feature. Full measurement equations are derived. It is shown, that is possible to provide three-dimensional update information from SAR measurements. The obtainable accuracy is given depending on various flight scenarios.

#### 1. INTRODUCTION

Nowadays navigation systems rely on different sensors to provide accurate and reliable navigation information. Most important is the inertial measurement unit (IMU) which typically consists of three accelerometers and tree gyroscopes to measure accelerations and angular rates along all tree axes. A strapdown algorithm integrates measurements in a proper way to propagate velocity, position and attitude information. However navigation accuracy decreases with time. Additional sensors have to be used to correct position and velocity estimates frequently to prevent unbounded growth of navigation errors.

Radar altimeter measurements are able to accurately update the navigation information in regions of sufficient rough terrain by a comparison of the measured terrain height and a stored height data map. Thus, the navigation solution is widely independent from external navigation signals e.g. GPS which can easily be jammed. However, radar altimeter measurements can not guarantee an aiding of the inertial navigation system during flights over smooth terrain. To avoid increasing navigation errors a synthetic aperture radar measurement step is proposed.

A synthetic aperture sensor images the earth surface in range and azimuth coordinates. The image is transformed into earth coordinates and matched to a map which contains specific well visible and unambiguous features like crossroads or courses of rivers. The position shift between the captured SAR image and the reference feature is used to aid the navigation information. Many matching techniques are described in literature. Thus, it is assumed that matching algorithms exist and that a match between the SAR image and the reference feature containing feature position as well as feature position accuracy can be achieved.

This paper concentrates on the processing of the detected position shift between the SAR image and the reference feature. It is shown, that is possible to provide three-dimensional update information from SAR measurements. Simulation results additionally show that a SAR measurement step leads to a more accurate and more reliable position estimate in regions of smooth terrain.

An error in height estimation leads to an error of the estimated feature position in a nonlinear way. Higher order terms in the measurement equation may become significant in the presence of height or velocity errors. Sigma-point Kalman filter (SPKF) takes higher order terms into consideration. Due to this fact a sigma-point Kalman filter was implemented.

First chapter 2 gives an overview of a SAR/INS system. Chapter 3 introduces the SAR sensor geometry and the measurement equations used for Kalman filtering in detail. Nonlinear Kalman filtering by sigma-point Kalman filter is described in chapter 4. Chaper 5 briefly sumarizes standard terrain referenced navigation (TRN) systems which could be combined to SAR/INS systems. Finally simulation results are given in section 6.

## 2. SYSTEM OVERVIEW

The desired navigation system fuses inertial measurement data with feature measurements from synthetic aperture radar capturing the earth's surface. A block diagram is shown in figure 1. The inertial measurement data is first integrated by a strapdown algorithm (SDA). The SDA provides position, velocity and attitude information by integrating acceleration measurements  $\vec{a}_{ib}^B$  und angular rate measurements  $\vec{\omega}_{ib}^B$  in body frame coordinates [1]. However, measurement errors of the inertial measurement unit (IMU) and the integration process of the SDA leads to an unbounded growing of the navigation errors over time. Additional update information extracted from SAR feature matching is used to avoid increasing navigations errors.

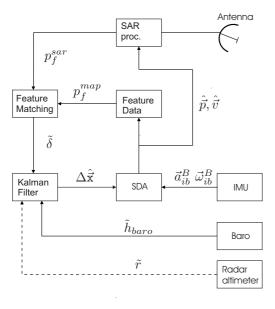


Figure 1. System

The synthetic aperture radar receives radar responses of the transmitted chirp pulses over a certain period of time. SAR raw data is then processed by a SAR processing algorithm to form a radar image. Hereby SAR processing depends on the knowledge of absolute velocity and position information from the SDA for range and azimuth compression as well as to transform the SAR image to ground coordinates.

SAR images are compared to the feature map data by a feature matching algorithm. Features which are expected to occur in the SAR image are selected from feature data and matched to the captured SAR image. Different SAR feature matching techniques are described in [2], [3] and [4]. The displacement is according to equation 1 expressed by the position difference of the map feature  $p_f^{map}$  and the feature captured by the SAR sensor  $p_f^{sar}$ . The displacement between map feature position and the position where the feature occurs in the SAR image is used to update the navigation information of the SDA by Kalman filtering.

(1) 
$$\delta = \begin{pmatrix} \delta_x \\ \delta_y \end{pmatrix} = \begin{pmatrix} p_{f,x}^{sar} - p_{f,x}^{map} \\ p_{f,y}^{sar} - p_{f,y}^{map} \end{pmatrix}$$

Map feature errors, inaccuracies of the alignment process, and navigation errors in position and velocity components lead to a position shift between measured and estimated feature position in a nonlinear way. This nonlinearity can be taken into consideration by sigma-point Kalman filtering to improve SAR/INS integration. Figure 2 shows an example of a feature with its displacement.

A barometric altimeter is integrated to the navigation system to aid height estimation.

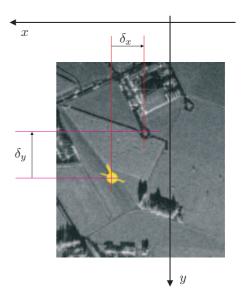


Figure 2. Feature displacement

Additionally a radar altimeter which measures distance form aircraft to the ground level leads to an improved navigation accuracy and a more reliable position estimation during flights over rough terrain.

## **3. SYNTHETIC APERTURE RADAR**

Synthetic aperture radar is able to provide two dimensional images of the earth surface independently of external sources and time of day due to active illumination of the scene. Additionally SAR imaging is nearly independent of weather influences which is its main advantage compared to optical systems.

In general the SAR principle means the collection of many radar responses of the transmitted chirp pulses by the radar antenna over a straight flight track at the pulse repetition frequency. By means of this raw data collection a larger synthetic aperture is formed which allows to achieve good resolution in azimuth direction. Range resolution is achieved by transmitting chirp pulses of a certain bandwidth.

A main disadvantage is the condition of a nearly straight flight path during the collection of the raw data. Therefore SAR snap shot mode is assumed that only integrates radar responses over a short flight path which can be assumed to be nearly straight. On the other hand azimuth resolution is decreased due to the shorter synthetic aperture but azimuth resolution is still sufficient for SAR feature matching.

#### 3.1. SENSOR COORDINATE FRAME

First a sensor coordinate frame called s-frame has to be defined. Feature displacement  $\delta$  is given in s-frame coordinates x and y.

The sensor frame coordinate system is shown in figure 3. The origin is located underneath the estimated aircraft position at the height level of the expected feature in the current sar image. The x-axis of the sensor coordinate system is parallel to the horizontal velocity of the aircraft,

z-axis points upwards and y-axis forms a right-handed coordinate system. Compared to navigation frame coordinates north, east and down the sensor coordinate is rotated by the angle  $\Psi_v$  which corresponds to the heading of the aircraft.

(2) 
$$\Psi_v = \arctan\left(\frac{\hat{v}_e^N}{\hat{v}_n^N}\right)$$

A transformation from navigation frame coordinate system to sensor frame coordinate system is done by using the transformation matrix  $C_N^S$  given in equation 3.

(3) 
$$C_N^S = \begin{pmatrix} \cos(\Psi_v) & \sin(\Psi_v) & 0\\ \sin(\Psi_v) & -\cos(\Psi_v) & 0\\ 0 & 0 & -1 \end{pmatrix}$$

Thus, the estimated velocity  $\hat{\vec{v}}^S$  is calculated from  $\hat{\vec{v}}^N$  according to equation 5. Estimated position in s-frame coordinates is given in equation 4. Due to the definition of the origin of the sensor frame coordinate system the position estimates in x and y direction always equal to zero.

(4) 
$$\hat{p}^S = \begin{pmatrix} 0 & 0 & \hat{p}_z \end{pmatrix}$$
  
(5)  $\hat{v}^S = \begin{pmatrix} \hat{v}_x & 0 & \hat{v}_z \end{pmatrix}^T = C_N^S \hat{v}^N$ 

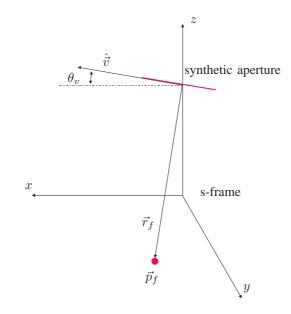


Figure 3. Geometry

The feature matching process and the measurement are done in s-frame coordinates. All measurement equations are derived in the following chapters in the s-frame coordinate system that is more descriptive than navigation frame coordinates in the case of SAR measurements.

# **3.2. SYNTHETIC APERTURE RADAR MEASURE-MENT**

Due to the short integration time of SAR snap shot mode it is possible to just consider the SAR geometry at the reference point which is the middle of the synthetic aperture of the current image and corresponds to the sframe position given in equation 4. A certain feature point is assumed to be located at  $\vec{p}_f$ , The vector from SAR antenna to the feature point then is expressed by

(6) 
$$\vec{r}_f = \begin{pmatrix} p_{f,x} & p_{f,y} & -p_z \end{pmatrix}$$

Synthetic aperture radar processing forms a radar image depending on the range and the Doppler information of the reflection points. The current feature point located at  $\vec{p}_f$  leads to a range and Doppler frequency given in equation 7 and 8.

$$(7) R = |\vec{r}_f|$$

(8) 
$$f_d = \frac{2 \cdot \vec{r}_f \cdot \vec{v}^S}{|\vec{r}_f| \cdot \lambda}$$

Doppler and range information within the radar raw data are used to restore the image by a SAR processing algorithm. Azimuth direction is determined by Doppler frequency information. The current feature point will occur at slant coordinates  $R_s$  and  $R_x$ , where  $R_x$  is along the aircraft velocity axes and  $R_s$  is the slant range perpendicular to the velocity axes.

(9) 
$$R_x = \frac{\vec{r}_f \cdot \vec{v}^S}{|\hat{\vec{v}}^s|}$$

$$(10) R_s = \sqrt{R^2 - R_x^2}$$

However, the SAR image has to be transformed into xycoordinates in sensor frame for alignment to the map data. Image transformation is done within the SAR processing block. With respect to the estimated height of the aircraft above the feature point the feature will occur in s-frame coordinates at

(11)  

$$p_{f,x}^{sar} = R_x \cdot \cos(\theta_v) + \tan(\theta_v) \cdot (p_z + R_x \cdot \sin(\theta_v))$$
(12)  

$$p_{f,y}^{sar} = \sqrt{R_s - \left(\frac{p_z}{\cos(\theta_v)} + R_x \cdot \tan(\theta_v)\right)^2}.$$

To calculate the position displacement of the feature point according to equation 1, the map feature position has to be known. Map feature position in sensor coordinate system is determined by the difference of the stored feature position and the current estimated position of the aircraft and a rotation into the desired s-frame coordinate system.  $\phi_f$  and  $\lambda_f$  denote the features longitude and latitude position,  $\phi$  and  $\lambda$  the aircraft position, respectively.  $R_n$  is the earth radius in north direction and  $R_e$  describes earth radius in east direction.

(13) 
$$p_f^{map} = C_N^S \cdot \begin{pmatrix} (\phi_f - \phi)/R_n \\ (\lambda_f - \lambda)/(R_e \cdot \cos(\Phi)) \\ 0 \end{pmatrix}$$

The measurement corresponds to the difference between the feature position in the SAR image and the estimated feature position from feature map data. Additionally the measurement is affected by horizontal and vertical inaccuracies of the map feature position and the uncertainty of the alignment process. These errors are represented by the measurement noise. Errors of the SAR sensor itself are assumed to be very small compared to map and alignment errors and are neglected. Thus, the measurement equation is written as

(14) 
$$\tilde{\delta} = \begin{pmatrix} \delta_x \\ \tilde{\delta}_y \end{pmatrix} = \begin{pmatrix} h_x(\vec{p}, \vec{v}, \nu_x) \\ h_y(\vec{p}, \vec{v}, \nu_y) \end{pmatrix}$$
$$= \begin{pmatrix} p_{f,x}^{sar} - p_{f,x}^{map} \\ p_{f,y}^{sar} - p_{f,y}^{map} \end{pmatrix} + \begin{pmatrix} \nu_{a,x} \\ \nu_{a,y} \end{pmatrix}$$

## 4. SIGMA-POINT KALMAN FILTER

A sigma-point Kalman filter in error state formulation has been implemented for SAR/INS integration. The state vector of the filter includes position, velocity, attitude und bias errors of the accelerometers and gyroscopes.

(15) 
$$\hat{\vec{x}} = \begin{pmatrix} \Delta \vec{p}^N & \Delta \vec{v}^N & \Delta \vec{\Psi} & \Delta \vec{b}_a & \Delta \vec{b}_\omega \end{pmatrix}$$

In opposite to the extended Kalman filter, the SPKP does not require the approximation of nonlinear measurement functions using the Jacobian to calculate the Kalman gain matrix. Instead, the probability density function of the Kalman filter state, which is assumed to be Gaussian, is replaced by a deterministically chosen discrete point density. These points are called sigma-points. In the measurement step, the sigma-points are transformed through the nonlinear measurement model. The mean, covariance and correlation are calculated directly from the transformed sigma-points, which ensures determination of mean and covariance accurately at least to the second order, while the EKF achieves first order accuracy only [5]. First an augmented state vector  $\vec{x}^a$  is constructed which includes the state vector and the measurement noise components. The dimension of this augmented state vector is denoted by L.

(16) 
$$\hat{\vec{\mathbf{x}}}^{\mathbf{a}} = E[\vec{\mathbf{x}}^{a}] = \left(\hat{\vec{\mathbf{x}}}^{T}\hat{\vec{\nu}}^{T}\right)^{T}$$
  
(17)  $\mathbf{P}^{\mathbf{a}} = E[(\hat{\vec{\mathbf{x}}}^{a} - \bar{\vec{\mathbf{x}}}^{a}) \cdot (\hat{\vec{\mathbf{x}}}^{a} - \bar{\vec{\mathbf{x}}}^{a})^{T}] = \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{pmatrix}$ 

The measurement noise includes horizontal errors  $\nu_{f,h}$ and vertical errors  $\nu_{f,v}$  of the reference feature and errors of the matching process in x and y direction  $\nu_{a,x}$  and  $\nu_{a,y}$ , respectively.

(18) 
$$\vec{\nu} = \begin{pmatrix} \nu_{f,h} & \nu_{f,h} & \nu_{f,v} & \nu_{a,x} & \nu_{a,y} \end{pmatrix}^T$$

The covariance of the augmented state vector is constructed form the filter state covariance and the measurement noise covariance.

(19) 
$$\mathbf{R} = \begin{pmatrix} \sigma_{f,h}^2 & 0 & 0 & 0 & 0\\ 0 & \sigma_{f,h}^2 & 0 & 0 & 0\\ 0 & 0 & \sigma_{f,v}^2 & 0 & 0\\ 0 & 0 & 0 & \sigma_{a,xx}^2 & \sigma_{a,xy}^2\\ 0 & 0 & 0 & \sigma_{a,xy}^2 & \sigma_{a,yy}^2 \end{pmatrix}$$

The measurement noise covariance again includes horizontal and vertical map feature errors  $\sigma_{f,v}$  and  $\sigma_{f,h}$  as well as alignment errors of the matching process  $\sigma_a$ 

A set of 2L+1 sigma-points is chosen to represent the state vector and its covariance. The scaling parameter  $\zeta$  determines the spread of the sigma-points and must be chosen with respect to the weighting factors, such that the point density possesses the same covariance. The square root of the augmented covariance matrix is calculated using the Cholesky decomposition. Each column of the square root forms a single sigma-point within the surrounding of the state vector.

(20) 
$$\vec{\chi}_0^a = \hat{\vec{x}}^a$$
  
(21)  $\vec{\chi}_i^a = \hat{\vec{x}}^a + \zeta \sqrt{\mathbf{P}^a}_i, \quad i = 1...L$   
(22)  $\vec{\chi}_{i+L}^a = \hat{\vec{x}}^a - \zeta \sqrt{\mathbf{P}^a}_i, \quad i = 1...L$ 

In the filter estimation step, all sigma-points are transformed by the measurement equation.

(23) 
$$\vec{Y}_i = \vec{h}(\vec{\chi}_i^a)$$

With this set of transformed sigma-points, mean, covariance and correlation between sigma-points and transformed sigma-points are calculated directly using the given formulas. Within the calculation of the correlation only the sigma-point components which represent the state vector are used instead of the augmented sigma-point vector.

(24) 
$$\hat{\delta} = \sum_{i=0}^{2L} w_i^m \vec{Y}_i$$
  
(25)  $\mathbf{P}_{\mathbf{yy}} = \sum_{i=0}^{2L} \sum_{j=0}^{2L} w_{ij}^c (\vec{Y}_i - \hat{\delta}) (\vec{Y}_j - \hat{\delta})$ 

(26) 
$$\mathbf{P_{xy}} = \sum_{i=0}^{2L} \sum_{j=0}^{2L} w_{ij}^c (\vec{\chi}_i - \hat{\vec{x}}) (\vec{Y}_j - \hat{\delta})$$

The knowledge of these quantities then allows the update of the state vector estimation as well as the state vector covariance by the Kalman filter equations.

(27) 
$$\mathbf{K} = \mathbf{P}_{\mathbf{x}\mathbf{y}}\mathbf{P}_{\mathbf{y}\mathbf{y}}^{-1}$$

(28) 
$$\hat{\vec{x}}^+ = \hat{\vec{x}}^- + \mathbf{K}(\tilde{\delta} - \hat{\delta})$$

(29) 
$$\mathbf{P}^+ = \mathbf{P}^- - \mathbf{K} \mathbf{P}_{\mathbf{v}\mathbf{v}} \mathbf{K}^T$$

These update equations of the sigma-point Kalman Filter are analogous to an extended Kalman filter update step. The difference is in calculation of covariance  $\mathbf{P}_{yy}$  and correlation  $\mathbf{P}_{xy}$  as well as mean  $\hat{\delta}$  which are accurate to the first order term in case of sigma-point Kalman filtering. Standard Kalman filter update step is given for comparison.

(30) 
$$\mathbf{K} = \mathbf{P}^{-}\mathbf{H}^{T} \cdot (\mathbf{H}\mathbf{P}^{-}\mathbf{H}^{T} + \mathbf{R})^{-1}$$
  
(31) 
$$\hat{\mathbf{x}}^{+} = \hat{\mathbf{x}}^{-} + \mathbf{K} \cdot (\hat{\delta} - \hat{\delta})$$
  
(32) 
$$\mathbf{P}^{+} = (\mathbf{I} - \mathbf{K}\mathbf{H}) \cdot \mathbf{P}^{-}$$

#### 5. TERRAIN REFERENCED NAVIGATION (TRN)

In a terrain referenced navigation navigation system a radar altimeter measures height of the aircraft above ground level. The radar height is used as approximation of the aircraft to the nadir point directly below the aircraft. This can be done because of the use of wide radar beam width. This ensures that even for a pitching or rolling aircraft the nadir point lies within footprint of the radar antenna.

The generation of position aiding is done by processing radar height measurements and a reference map which contains terrain elevations at certain spacing along latitude and longitude coordinates. For terrain with reasonable roughness the radar height measurements can be compared to the reference map to update vertical as well as horizontal position. During flights over flat terrain where terrain roughness is low compared to the radar noise level the horizontal position cannot be identified.

Standard terrain referenced navigation with nadir point height measurements and a sequential processing is combined to the SAR/INS system.

(33) 
$$\tilde{r} = -h - h_{map}(\phi, \lambda) + \nu$$

$\tilde{r}$	height over ground measurement
h	absolute height in down coordinates
$h_{map}$	terrain height
$\phi,\lambda^{}$	latitude and longitude
ν	measurement noise

To process radar measurement data extended Kalman filter has been used. The Jakobian matrix is given in equation 34. The first derivatives of the reference map with respect to north and east position cannot be calculated analytically. The derivatives are approximated using the discrete height data of the reference map.

(34) 
$$\mathbf{H} = \left(\frac{\partial h_{map}}{\partial x_n}, \frac{\partial h_{map}}{\partial x_e}, -1, 0, \ldots\right)$$

Following, extended Kalman filter update step can be processed.  $\hat{r}$  denotes the expectation of the current radar height measurement.

(35) 
$$\mathbf{K} = \mathbf{P}^{-}\mathbf{H}^{T} \cdot (\mathbf{H}\mathbf{P}^{-}\mathbf{H}^{T} + \mathbf{R})^{-1}$$

(36) 
$$\hat{\vec{x}}^+ = \hat{\vec{x}}^- + \mathbf{K} \cdot (\hat{\mathbf{r}} - \hat{\mathbf{r}})$$

$$(37) \mathbf{P}^+ = (\mathbf{I} - \mathbf{K}\mathbf{H}) \cdot \mathbf{P}^-$$

Standard terrain referenced navigation system with nadir point measurements or directional measurements is explained in [6] and [7].

#### 6. SIMULATION RESULTS

Various flight scenarios has been simulated to analyse SAR/INS position accuracy. Additionally a combination of SAR/INS with standard terrain referenced navigation has been implemented using the nadir model [6].

## 6.1. ERROR CHARACTERISTIC OF SAR/INS SYS-TEMS

Figure 4 shows the position error at different SAR feature update rates using a navigation grade IMU. As expected the navigation errors will become smaller if feature points occur more often. In practise update rates will depend on the occurrence of unambiguous and well visible features. As shown in figure 4 a feature update interval in the scale of a few minutes is required.

On the other hand navigation solution also depends on the position of the features relative to the aircraft. To ensure accurate positioning of the aircraft one needs mathematically independent measurements. This can be achieved by features at different depression angles. This leads to a different geometry and therefore to mathematically independent measurements.

SAR feature updates are modelled as point fixes with measurement errors specified below. Map errors are assumed to be 3 meters rms in horizontal and vertical direction. Additionally it is assumed that feature matching can be done to an accuracy of 7 meters which typically corresponds to the width of a street segment used as feature.

	errors
$\sigma_a = 7m$	Standard deviation of matching
	map errors
$\sigma_{f,v} = 3m$	Standard deviation of vertical
	tal map errors
$\sigma_{f,h} = 3m$	Standard deviation of horizon-

Figure 5 and figure 6 show velocity error and attitude error, respectively. Velocity also depends on the feature update rate, whereas attitude error is relatively independent. It is shown, that it is possible to aid position as well as velocity estimation three dimensionally by SAR/INS systems. Height estimation is critical in SAR/INS systems. Therefore barometric altimeter measurements are used additionally in all simulation runs.

## 6.2. SAR/INS in combination with terrain referenced navigation

Figure 7 shows simulation results of the desired flight using TRN updates only and SAR/TRN with feature updates each 120 seconds. Due to the extreme smoothness of the terrain underneath the aircraft at the end of the flight TRN cannot deliver reliable position information. SAR feature measurements avoid divergence of the filter and ensure accurate horizontal position estimation. On the other hand TRN measurements decrease vertical position errors compared to a SAR/INS system. TRN and SAR show different characteristics such that both measurements profit from each other. Figure 8 shows simulation results of a SAR/TRN combination in comparison to TRN updates only. Maximum position errors before a next feature detection are decreased in case of reasonable rough terrain.

Radar altimeter measurements ensure accurate height information and are able to autonomously aid the navigation solution in regions of rough terrain by comparing terrain height measurements to a reference height map whereas SAR prevents increasing navigation errors over smooth terrain, where manmade features like crossroads commonly occur more often.

The combination of SAR/INS and TRN turned out to be optimal due to the different sensor characteristics. The navigation system does not rely on TRN during flights over smooth terrain but on SAR updates. Therefore, less accurate TRN systems are required in a combined TRN/SAR/INS navigation system. Low cost radar altimeter and standard DTED level 1 height data is suitable.

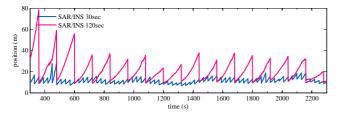


Figure 4. Position error at different feature detection rates

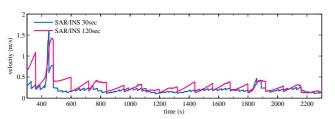


Figure 5. Velocity error at different feature detection rates

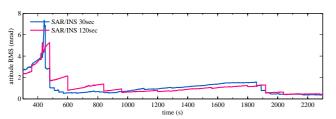


Figure 6. Attitude error at different feature detection rates

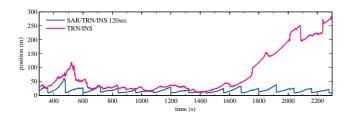


Figure 7. Position error of TRN/INS and TRN/INS in combination with SAR  $% \left( {{\rm SAR}} \right) = {\rm SAR}$ 

## 7. CONCLUSION

Simulation results show that is possible to aid position by integrating SAR measurements using sigma-point

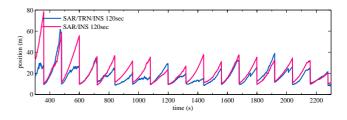


Figure 8. Position error of SAR/INS and SAR/TRN/INS

Kalman filter. Feature detections each two minutes still ensure stable position estimation.

The combination of SAR/INS and even low cost TRN is optimal due to the complementary characteristics. TRN aids the position during flights over regions of reasonable rough terrain whereas SAR prevents increasing position errors over smooth terrain where manmade features e.g. crossroads occur more often.

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