Estimation of the TOA performance of Loran-C

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Abstract

Loran-C is a navigational aid that relies on the ability to make correct estimates of the *Times of Arrival* (TOAs) of signals received over a noisy radio channel. How good is the performance of Loran-C in this respect? Is it close to optimal? Are there perhaps better estimation techniques available than we currently employ? In this paper we use *Estimation Theory* to find the optimal limit of the accuracy of TOA estimates, for a given *signal-to-noise ratio* (SNR). Specifically, we compute the *Cramer-Rao Lower Bound* (CRLB) which specifies the absolute minimum error variance that can be achieved by an unbiased estimator. From the CRLB we can estimate the best repeatable accuracy that can be attained using a set of TOAs. We compute here the CRLBs for the TOA of a Loran-C signal with both *Additive White Gaussian Noise* (AWGN) and *Additive Coloured Gaussian Noise* (ACGN). We then employ the *Maximum Likelihood Estimator* (MLE) and compare its results with that of CRLB. Simulation results show that the MLE approaches the minimum variance obtained by CRLB for SNR values of practical and reliable Loran-C operation.

1 Introduction

Loran-C is a long-range radio navigation aid in which, in its original form, the user established a position by measuring the time differences between pairs of signals received from synchronised terrestrial transmitters. A hyperbolic method was then used to compute a position in two dimensions. Now, it is chiefly used in an “all-in-view” mode in which the times of arrival (TOAs) of signals received from multiple stations are measured against an on-board clock. A position is then calculated using a weighted best-fit computation, together with the clock error. Loran transmitting stations are typically spaced 500-1000 km apart and their locations are known very precisely. Details of the relatively complex Loran transmissions, at their frequency of 100 kHz, are given in [1-4].

The repeatable accuracy [5] with which position fixes are made is usually expressed as a “2drms” value, computed from the variances of the estimates of the contributing TOAs. Thus, to estimate the highest repeatable accuracy, we need to know the lower bound of the variances of the TOA measurements. There are several standard ways of estimating such variance bounds, the most common (and the easiest to calculate) being the *Cramer-Rao lower bound* (CRLB) [6, 7]. The CRLB is an estimate of the minimum variance attainable by an unbiased estimator, for a given *signal-to-noise ratio* (SNR).

In this paper, we will calculate the optimum *Time of Arrival* (TOA) performance of the Loran-C system by estimating the CRLB for the position error. We will also explore the use of *Maximum Likelihood Estimation* (MLE) and compare its performance with CRLB.

Since the CRLB is dependent on the noise present, we conduct simulations and calculate the CRLB using two different kinds of noise. First, we assume that there is thermal noise only present in the system; we represent this by using *Additive White Gaussian Noise* (AWGN). A more realistic kind of noise is *Additive Coloured Gaussian Noise* (ACGN) as a result of atmospheric noise and the front-end bandpass filtering at the receiver. We will also evaluate the performance of the proposed methods for different SNR levels and observation times.

The paper is structured as follows. Section 2 presents a brief description of the Loran-C signal and the noise used in this analysis. In Section 3, an extended overview of the theoretical background to the CRLB calculation and the MLE principle is provided. In Section 4, we present the results of computer simulations that show the CRLB and MLE performance. Section 5 summarises the paper and discusses future research possibilities.

2 Signal Model

2.1 The Loran-C Signal

The Loran-C transmission consists of groups of pulses as described in [1]. The shape of each pulse is defined as:

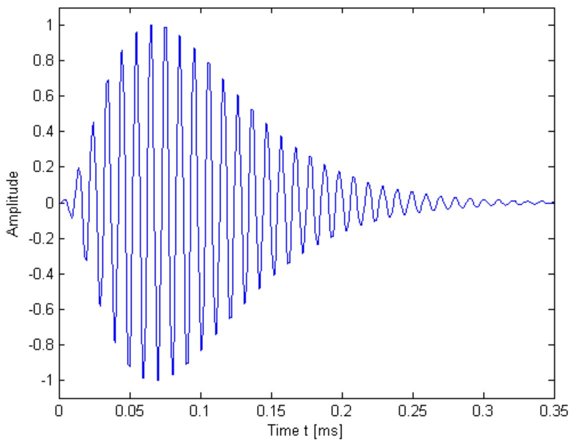
$$P(t) = \begin{cases} A(t-\tau_0)^2 \cdot e^{-\frac{2(t-\tau_0)}{65}} \cdot \mathrm{sgn}(t-\tau_0) & ; \quad \tau_0 - PRI/2 \le t \le 65+\tau_0 \\ \text{not defined} & ; \quad 65+\tau_0 < t \le \tau_0 + PRI/2 \end{cases} \tag{1}$$

where PRI is the *pulse repetition interval*. This envelope, P(t), amplitude modulates the 100 kHz carrier, so producing pulses of the form shown in Fig. 1, and described by:

$$s(t) = P(t) \cdot \sin(2\pi f_c t + \phi(t)) . \tag{2}$$

Loran-C pulses are transmitted in groups of 9 by each *master* station, and 8 by each *secondary*. The phases of the carrier in the individual pulses are coded as specified in [1]. The *Group Repetition Interval* (GRI) of each chain of Loran stations is between 40 and 100 ms. To calculate a position by the traditional, hyperbolic, method the receiver measures the differences of the times of arrival of pulses from each of two secondary stations of a chain with respect to the pulses received from the master station.

Transforming the description of the Loran-C pulse so that we see its frequency domain representation allows us to make certain simplifications which are of benefit when we compute the CRLB as will be shown in Section 3. We obtain the frequency domain Power Spectrum S(f) of the Loran-C signal by calculating the magnitude of the squared *Fourier transform* of its time domain signal, s(t). The resulting power spectrum is shown, in normalised form, in Fig. 1.



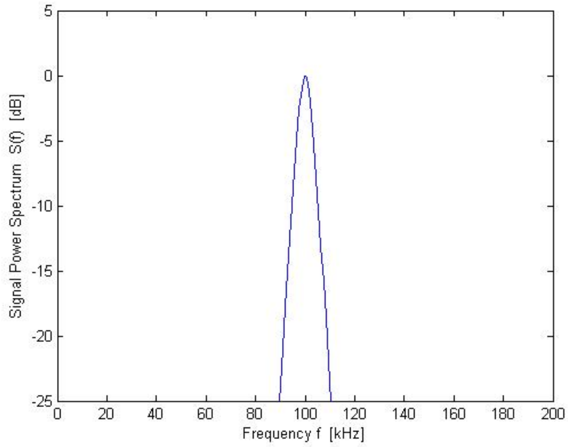


Fig. 1: Loran-C signal. (Left) Time-Domain representation, s(t), and (Right) Frequency-Domain representation, S(f).

2.2 Atmospheric Noise

Since the CRLB is dependent on the noise present (as will be shown in Section 3), we conduct simulations and calculate the CRLB using two different kinds of noise. First, we assume that there is thermal noise only present in the system; we represent this by using *Additive White Gaussian Noise* (AWGN). Unfortunately, the white noise assumption is never true in practice since the signals are subject to atmospheric noise and front-end filtering at the receiver. This type of noise is called *Additive Coloured Gaussian Noise* (ACGN).

Atmospheric noise is the dominant noise in the Loran-C band. Atmospheric noise in the low frequency (LF) band consists essentially of coloured Gaussian noise produced by a large number of separate lightning discharges which are received via skywave propagation, plus a smaller number of strong discharges caused by lightning at shorter ranges which are received via groundwave propagation. In this paper we use *Atmospheric Simulated Random Noise* (ASRN), which is generated in accordance with the standard defined in the Loran-C *Minimum Performance Standards* (MPS) of The Radio Technical Commission for Marine Services [2]. We propose to employ Matlab for our simulations. ASRN will be generated by creating random noise and passing it through a serial LC filter specified in the MPS [2]. The resulting output is then transformed from the analogue to the digital domain by means of a bilinear transformation; the spectrum of this filter is shown in Fig. 2.

2.3 The Receiver Front-end Filter

In a Loran receiver, the Loran-C signal and the atmospheric noise pass through a front-end bandpass filter. We simulated this by means of a 5^{th} order Butterworth filter with a bandwidth of 20 kHz. The specific filter used is one employed by the North West European Loran-C Technical Working Group in predicting the coverage of Loran-C chains. Fig. 2 (left) shows the magnitude responses of the frequency domain transfer functions of this receiver filter and of the noise LC-filter presented in Section 2.2. Fig. 2 (right) shows the spectrum of the ASRN after it has passed through this front-end filter.

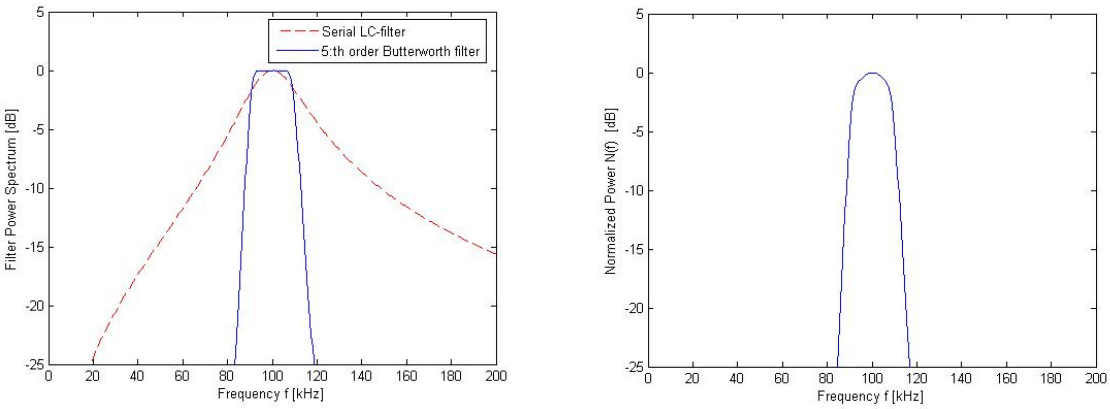


Fig. 2: (Left) Magnitude responses of typical receiver front-end filter (blue) and an LC- filter used to create ASRN (red). (Right) Power Spectrum N(f) of ASRN after passing through receiver front-end filter.

3 Applying Estimation Theory to Loran TOAs

In this section we employ the principles of Estimation and Information Theories with the aim of obtaining an optimal bound for the TOA in Loran-C system. As we have shown in Section 2 (Figs. 1 and 2) that the spectrums are presented in terms of the frequency f in Hertz [Hz]. However, when calculations of CRLB are done it is easier to use the angular frequency ω in radians per second [rad/s] rather than the absolute frequency as will be presented in Section 3.2.

3.1 Measurement Uncertainties of Physical Systems and the Fisher Information

Measurements of physical properties are subject to uncertainties or errors. In measuring the times of arrival of Loran-C signals, we may experience uncertainties caused by thermal movement of electrons, atmospheric disturbances, skywave interference, interfering electromagnetic fields generated by power lines, electric machinery or cosmic events. If we avoid or remove all interference, the dominant form of noise by many orders of magnitude is atmospheric noise. We can represent the sum of all noise and interference received by the random variable n(t). Thus, if the noise-free signal is $s(t-\tau_0)$, the received signal with noise is x(t), where:

$$x(t) = s(t - \tau_0) + n(t) \tag{3}$$

Since the signal n(t) is time-varying and random, we have no means to predict its instantaneous value. However, we can say that there is a certain probability $p(x;\tau_0)$ that the signal will have a certain value at a certain time τ_0. The distribution of this probability will depend on the type of interference and noise present. If the distribution (probability density function) is Gaussian (Fig. 3), we can describe it by its mean, standard deviation and variance.

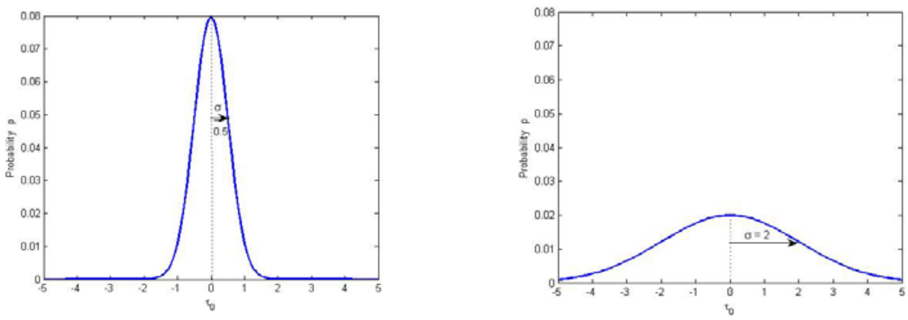


Fig. 3: Relationship between σ (the standard deviation) and the "sharpness" of the probability function using a Gaussian normal probability function.

Looking at Fig. 3, we can see that the *sharpness* of the curve of the probability density function can be measured by its *curvature*, which is defined as the rate of change of its tangent. If the tangent rate of change is "high" it means that the function is "sharp" and vice versa. The tangent of the probability function is the derivative of the function $p(x;\tau_0)$ and the rate of change of the tangent is, consequently, the second derivative of the function $p(x;\tau_0)$. Thus, by taking the average value of the second derivative of the probability function $p(x;\tau_0)$ we obtain a measure of how much fluctuations (or total noise and interference) that is added to the true value of the signal parameter (τ_0) that is being estimated. It is also convenient to use the natural logarithm of the probability function (i.e., $\ln p(x;\tau_0)$) rather than the probability function itself to ease the mathematical manipulations, since many of the existing probability functions contain exponential terms. If we write this in mathematical form we get the following equation:

$$I(\hat{\tau}_0) = -E\left\{\frac{\partial^2 \ln(p(x;\tau_0))}{\partial {\tau_0}^2}\right\} \tag{4}$$

where E denotes the expected value (or the average value). The variable $I(\hat{\tau}_0)$ is known as the *Fisher Information* which was developed by the statistician and geneticist R.A. Fisher in the 1920's [8]. If $I(\hat{\tau}_0)$ have a high value, then this means the probability function is narrow and we have small random fluctuations, which in turn means the accuracy of the time delay estimation $\hat{\tau}_0$ is high. On the other hand, if $I(\hat{\tau}_0)$ is low then this means the probability function is wide and thus we have high random fluctuations, which can be interpreted as the accuracy of the time delay estimation $\hat{\tau}_0$ being low. The Fisher Information $I(\hat{\tau}_0)$ is therefore the maximum available information or degree of accuracy that can be attained for a specific estimation.

3.2 Relationship between Fisher Information and the Cramer-Rao Lower Bound

In this section we explore the relationship between the Fisher Information and the statistical parameters of the system with the aim of deriving an expression of the optimal bound for the estimation accuracy. The variance σ^2 (or noise power) of the estimated time delay $\hat{\tau}_0$ is defined as the squared width of the probability function (see Fig. 3) or the square value of the standard deviation in mathematical terms. Thus, the minimum attainable variance of an estimate can be expressed as the *inverse* of the Fisher Information. This minimum variance forms a lower bound on the estimation variance and, consequently, on the accuracy of the estimation; this is called the *Cramer-Rao Lower Bound* (CRLB) [6, 7] and is given by:

$$\mathrm{var}(\hat{\tau}_0) \geq \frac{1}{I(\hat{\tau}_0)} = \frac{1}{-E\left\{\frac{\partial^2 \ln(p(x;\tau_0))}{\partial {\tau_0}^2}\right\}} \tag{5}$$

If we now consider the time difference of arrival estimation problem with reference to a traditional Loran-C receiver, we have two received signals $x_1(t)$ and $x_2(t)$ (these signals could be the received signals from a *master* station and a *slave* station, respectively) which can be described as:

$$
\begin{aligned}
x_1(t) &= s(t) + n_1(t) \qquad & -T/2 \le t \le T/2 \\
x_2(t) &= s(t-\tau_0) + n_2(t-\tau_0) \qquad & -T/2 \le t \le T/2
\end{aligned} \tag{6}
$$

where *s(t)* is the Loran-C pulse while $n_1(t)$ and $n_2(t)$ are mutually uncorrelated, weak stationary additive Gaussian noise processes. T is the observation time interval. To calculate the accuracy of the time difference τ_0, we can now use equation (5) to find the minimum variance and, consequently, the accuracy of the estimated time difference $\hat{\tau}_0$.

The calculation of the Fisher Information in accordance with equation (4), however, involves the need of inverting a large covariance matrix which is computationally inefficient, especially if the observation time T is large. The number of elements in this covariance matrix would be the squared number of samples taken during the observation time T. It is known from [9] and others that an approximation of the Fisher information $\mathrm{I}(\hat{\tau}_0)$ (known as the asymptotic CRLB) is often a more practical way of calculating the CRLB. For the approximated bound to be sufficiently good, the observation time T must be much longer than the correlation depth of the signal *s*(*t*), i.e. $T >> \tau_{corr.}$. The correlation depth τ_{corr} is defined as the time difference between the maximum value of the auto-covariance signal of s(t), and the time at which the auto-covariance signal of s(t) has subsided sufficiently close to zero. If this condition is met we can rewrite the Fisher Information in equation (4) in the *frequency* domain as [10]:

$$
\mathrm{I}(\hat{\tau}_0) = \frac{T}{4\pi}\int_0^{2\pi} trace\left\{\frac{\partial \mathbf{P}(\omega)}{\partial \tau_\mathbf{0}}\mathbf{P}^{-1}(\omega)\right\}^2 d\omega \tag{7}
$$

For more information on how this equation is derived, interested readers are referred to appendix 3D in [10]. In order to calculate the Fisher Information from equation (7) we first have to calculate the spectral density matrix $\mathbf{P}(\omega)$. Using the signals from equation (6), the auto-covariance of $x_1(t)$ can be written as:

$$
R_{11}(\tau) = E\{x_1(t)x_1(t-\tau_0)\} = E\{(s(t)+n_1(t))\,(s(t-\tau_0)+n_1(t-\tau_0))\} \tag{8}
$$

Assuming that the signal *s(t)* and the noise $n_1(t)$ are mutually uncorrelated, Fourier transforming the auto-covariance $R_{11}(\tau)$ will give the auto-spectral density $P_{11}(\omega)$:

$$
P_{11}(\omega) = \mathrm{F}\{R_{11}(\tau)\} = S(\omega) + N_1(\omega) \tag{9}
$$

where F is the Fourier transform operator. Performing the same operation on the auto-covariance of $x_2(t)$ and the cross-covariances of $x_1(t)$ and $x_2(t)$ will result in the auto-spectral density $P_{22}(\omega)$ and the cross-spectral densities $P_{12}(\omega)$ and $P_{21}(\omega)$, respectively. Using matrix notation these spectral densities form a spectral density matrix $\mathbf{P}(\omega)$:

$$
\mathbf{P}(\omega) = \begin{bmatrix} S(\omega)+N_1(\omega) & S(\omega)e^{j\omega\tau_0} \\ S(\omega)e^{-j\omega\tau_0} & S(\omega)+N_2(\omega) \end{bmatrix} \tag{10}
$$

By differentiating $\mathbf{P}(\omega)$ with respect to the time delay τ_0, then multiplying with the inverse of the spectral density matrix $\mathbf{P}^{-1}(\omega)$ and finally squaring the result we end up with the following expression:

$$
\left\{\frac{\partial \mathbf{P}(\omega)}{\partial \tau_\mathbf{0}}\mathbf{P}^{-1}(\omega)\right\}^2 = \frac{1}{\det(\mathbf{P}(\omega))}\begin{bmatrix} \omega^2 S^2(\omega) & 0 \\ 0 & \omega^2 S^2(\omega) \end{bmatrix} \tag{11}
$$

where $\det(\mathbf{P}(\omega))$ is the determinant of the spectral density matrix in equation (10). Taking the trace of the matrix in equation (11) will produce the following expression:

$$
trace\left\{\frac{\partial \mathbf{P}(\omega)}{\partial \tau_\mathbf{0}}\mathbf{P}^{-1}(\omega)\right\}^2 = \frac{2\omega^2 S^2(\omega)}{S(\omega)N_2(\omega)+N_1(\omega)S(\omega)+N_1(\omega)N_2(\omega)} = \frac{2\omega^2\cdot SNR_1(\omega)\cdot SNR_2(\omega)}{SNR_1(\omega)+SNR_2(\omega)+1} \tag{12}
$$

The Fisher information for the estimated time difference $\hat{\tau}_0$ according to equation (7) can be calculated by inserting equation (12) into equation (7). The Cramer-Rao lower bound can then be calculated using equation (5) which will give the following result:

$$\mathrm{var}(\hat{\tau}_0) \geq \frac{2\pi}{T\int_0^{2\pi}\left\{\omega^2 \frac{SNR_1(\omega)\cdot SNR_2(\omega)}{SNR_1(\omega)+SNR_2(\omega)+1}\right\}d\omega} \tag{13}$$

As can be seen from equation (13) the value of the bound is dependent on the signal-to-noise ratio (SNR) and the observation time T. If the noise used in these calculations is AWGN and the signal is assumed to have a boxlike spectrum and $SNR_1 = SNR_2$ over the whole signal bandwidth, then equation (13) would reduce into:

$$\mathrm{var}(\hat{\tau}_0) \geq \frac{2\pi}{T\int_{\omega_1}^{\omega_2}\left\{\omega^2 \frac{SNR^2}{1+2\cdot SNR}\right\}d\omega} \tag{14}$$

If we instead use the spectrum N(ω) of the ACGN produced by the ASRN model and front-end bandpass filtering (Fig. 2) and the spectrum of the Loran-C signal S(ω) (Fig.1), we can calculate the frequency dependent signal-to-noise ratio SNR(ω) = S(ω)/N(ω) used in equation (13) and is shown in Fig. 4.

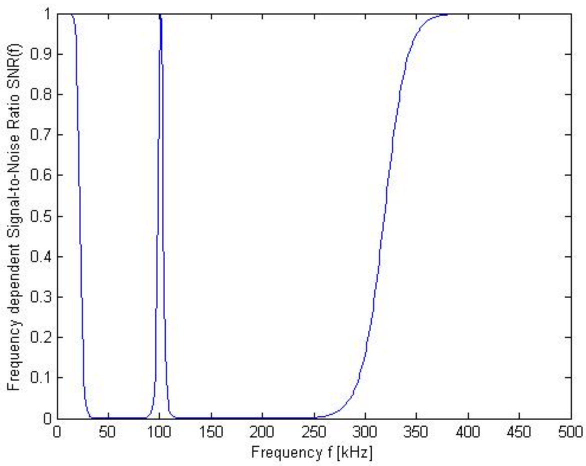


Fig. 4: The instantaneous frequency dependent signal-to-noise ration SNR(f) = $SNR\left(\frac{\omega}{2\pi}\right)$.

By using the definition of SNR(ω) function in equation (13) we can calculate the CRLB for the time difference τ_0 in a Loran-C system with an ACGN model. These results will be shown in the computer simulations presented in Section 4.

3.3 The Maximum Likelihood Estimator

The *Maximum Likelihood Estimator* (MLE) [9, 10] is the most used technique for parameter estimation. This is not only because of its close linkage to the Cramer-Roa Bound, but also because of its desirable asymptotic properties in the context on any problem. By calculating the MLE for our parameter, we are trying to find the most likely value of that parameter. MLE assumes that the probability function of the time delay parameter is known a *priori*. In our case we have a Gaussian distribution since the noise is modelled as an ACGN as describe earlier. The mathematical description of this probability function is:

$$p(\mathbf{x};\tau_0) = \prod_{n=0}^{N} \frac{1}{2\pi\sqrt{\det(\mathbf{C})}} e^{-\frac{1}{2}(\mathbf{x}_1(t)-\mathbf{x}_2(t-\tau_0))^{\mathbf{T}}\mathbf{C}^{-1}(\mathbf{x}_1(t)-\mathbf{x}_2(t-\tau_0))} \tag{15}$$

where $x_1(t)$ and $x_2(t)$ are the received signals as defined in equation (6), **C** is the covariance matrix and $s(t-\tau_0)$ is the signal at the sought after time τ_0.

Equation (15) describe the probability of finding a matching signal between $x_1(t)$ and $x_2(t)$ at the estimated time $\hat{\tau}_0$ [10, 11]. By maximizing this probability we will find the time $\hat{\tau}_0$ at which we most likely get the signal match. To ease the math, equation (15) may be simplified by using the natural logarithm of the probability function $\ln(p(x;\tau_0))$. To find the maximum of the function we differentiate it and set the result equal to zero. Thus, the maximum likelihood estimate (MLE) can be written as:

$$\frac{\partial \ln(p(\mathbf{x};\tau_0))}{\partial \tau_0} = -\sum_{t=0}^{T} -\mathbf{x}_1(t)\mathbf{x}_2(t-\tau_0) + \mathbf{x}_2(t-\tau_0)\mathbf{x}_2(t-\tau_0) = 0 \qquad (16)$$

Since $\mathbf{x}_2^2(t-\tau_0) = \mathbf{x}_2^2(t)$ and therefore not dependent on τ_0, the MLE in equation (16) can be simplified into:

$$\hat{\tau}_0 = \max_t \sum_{t=0}^{T} \mathbf{x}_1(t)\mathbf{x}_2(t-\tau_0) \qquad (17)$$

It is clear from equation (17) that the MLE of the time delay is the time lag at which the cross-correlation function between the received signals $x_1(t)$ and $x_2(t)$ has its maximum value. The MLE method will also be simulated and its results compared with that obtained from CRLB.

4 Simulation Results

In this section we present the simulation results and performance comparison between the methods presented in Section 3. The first simulation calculates the *Cramer-Rao Lower Bound* (CRLB) according to equation (13) using the frequency dependent SNR(ω) shown in Fig. 4 and an observation time T = 1 ms. The plot in Fig. 5 shows, in blue, the CRLB calculated using the coloured noise model in equation (13) and compared with the white Gaussian noise model (in red) using equation (14). The CRLB, as was shown in equation (5), is the minimum attainable variance $\sigma^2_{\hat{\tau}_0}$ of the estimated time difference $\hat{\tau}_0$. Thus, the maximum attainable accuracy in distance (range deviation σ_r in Fig. 5), is calculated by multiplying the square root of the variance of the estimated time difference with the speed of light. It is clear from this figure the deviation of CRLB results with coloured noise at high SNR values as compared with the ideal assumption of white noise usually used for calculating the bound.

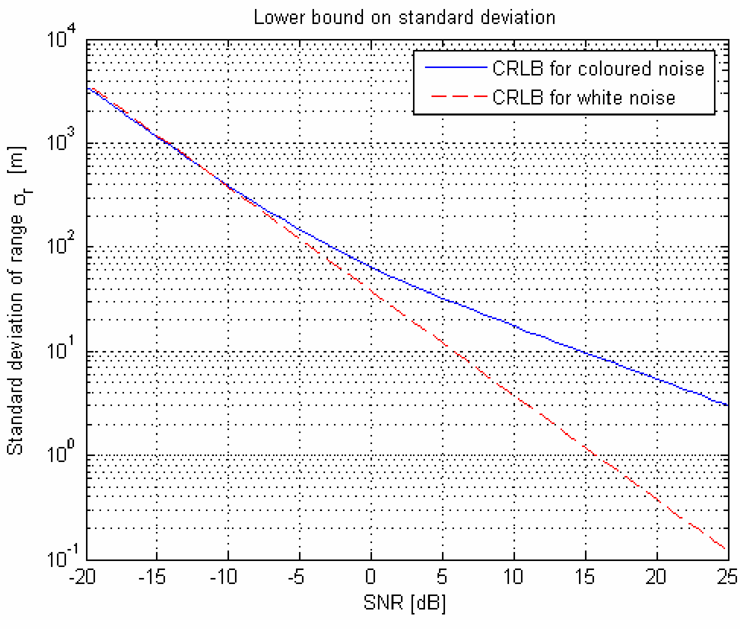


Fig. 5: CRLB with AWGN (red) and ACGN (blue) for an observation time of 1 ms. Note the difference in performance at high SNR values.

From equation (13) we can also see that the observation time T is an important factor when calculating the CRLB. Fig. 6 show the CRLB for various observation times ranging from 1 ms to 10 s, respectively. It is evident from this figure that the longer observation time we use the lower variance we get in the estimation process. This will, consequently, increase the accuracy of the position estimation.

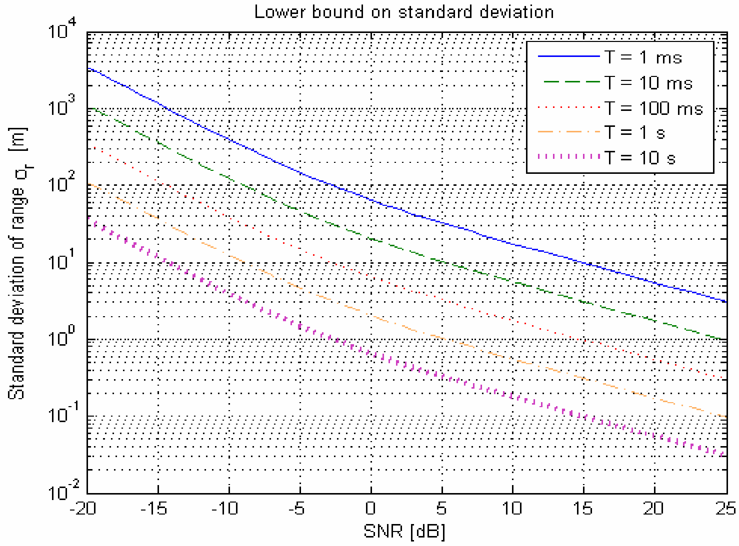


Fig. 6: The effect of observation time on the CRLB estimate. By increasing the observation time T the CRLB is improved.

Finally, Fig. 7 shows the results of *Maximum Likelihood Estimation* (MLE) for a simulated Loran-C signal with coloured noise model. The MLE is calculated according to equation (17) with an observation time of 1 ms and compared with the result obtained from CRLB. As can be seen from Fig. 7, the CRLB is overly optimistic since it does not conform very well to the actual behaviour of a detected signal, when considering estimations at low SNRs. The reason for this is that when the noise level is high, the MLE detector can not correctly choose the carrier wave cycle (wrong cycle selection or cycle skipping) when estimating the time difference. This might lead to a conclusion that it would be possible to reach a much higher accuracy than is practically possible in reality. However, it should be stressed that the CRLB defines the theoretical lower limit of signal measurement accuracy as determined solely by the characteristics of the signal (bandwidth, transmission time and received SNR). The bound does not factor in the real, physical effects of actual antennas, analog components and environmental anomalies. In addition, the results produced at these low SNR values are not of practical importance for reliable Loran-C operation. For example, for an SNR of 20 dB (-17 dB antenna) and higher, it is clear from Fig. 7 that the MLE is converging towards the theoretical lower limit of CRLB.

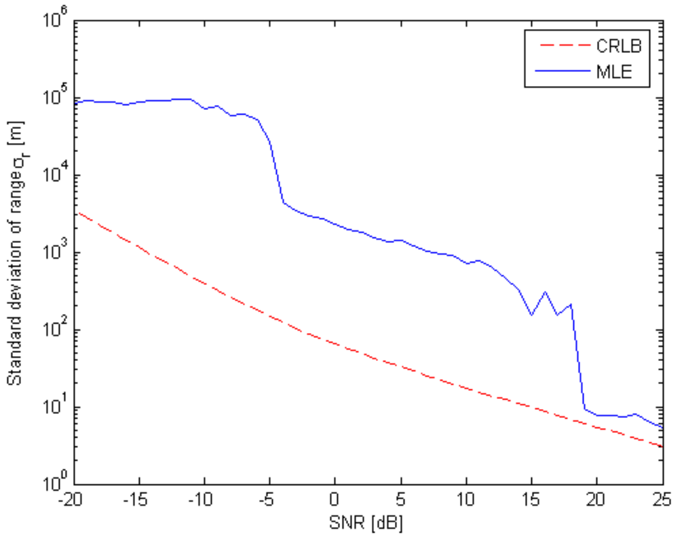


Fig. 7: The Maximum Likelihood Estimation converging towards the CRLB at high SNRs.

5 Conclusions and future work

In this paper, the optimum Times of Arrival (TOA) performance of the Loran-C system was analysed and investigated. This was achieved by calculating the Cramer-Roa Lower Bound (CRLB) for the minimum variance position error. The Maximum Likelihood Estimator (MLE) was also investigated and compared with the results obtained by CRLB. Both techniques have also been tested using AWGN and ACGN which also contains atmospheric noise simulated in accordance with Loran-C Minimum Performance Standards. Simulation results have shown that the MLE approaches the minimum variance obtained by CRLB for SNR values of practical and reliable Loran-C operation. Up to the authors knowledge this is the *first time* the application of CRLB has been done for Loran-C. In addition, while the ideal white noise assumption is usually used for the calculation of the bound, this paper has extended the application to coloured noise. In future work it would be interesting to compare this CRLB and MLE with real off-air data and current Loran-C receivers in order to get full assessment of the performance.

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Biographies

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Abbas Mohammed is an Associate Professor at the Department of Signal Processing, Blekinge Institute of Technology, Sweden. He was awarded the PhD degree from the University of Liverpool, UK, in 1992 and the Swedish "*Docent degree*" (equivalent to 2 PhD degrees in different fields of research) in *Radio Communications and Navigation* from Blekinge Institute of Technology in 2001. From 1993 to 1996 he was a Post-Doctoral Research Fellow within the Radio Navigation Group at the University of Wales, Bangor, UK. He is a Fellow of IEE, life-member of International Loran Association, member of IEEE and IEICE, and an Associate Fellow of the UK's Royal Institute of Navigation. He is a Board Member of the IEEE Signal Processing Swedish Chapter. He is also nominated as a regular member of the Editorial Board of Radio Engineering Journal. He has published many papers on telecommunications and navigation systems. He has also developed techniques for measuring skywave delays in Loran-C receivers. He was the Editor of a special issue "Advances in Signal Processing for Mobile Communication Systems" of Wiley's International Journal of Adaptive Control and Signal Processing.